An Intertemporal CAPM with Stochastic Volatility

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Abstract

This paper studies the pricing of volatility risk using the first-order conditions of a long-term equity investor who is content to hold the aggregate equity market rather than tilting towards value stocks and other equity portfolios that are attractive to short-term investors. We show that a conservative long-term investor will avoid such tilts in order to hedge against two types of deterioration in investment opportunities: declining expected stock returns, and increasing volatility. Empirically, we present novel evidence that low-frequency movements in equity volatility, tied to the default spread, are priced in the cross-section of stock returns.

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1 Introduction

The fundamental insight of intertemporal asset pricing theory is that long-term investors should care just as much about the returns they earn on their invested wealth as about the level of that wealth. In a simple model with a constant rate of return, for example, the sustainable level of consumption is the return on wealth multiplied by the level of wealth, and both terms in this product are equally important. In a more realistic model with time-varying investment opportunities, long-term investors with relative risk aversion greater than one (conservative long-term investors) will seek to hold "intertemporal hedges", assets that perform well when investment opportunities deteriorate. Merton's (1973) intertemporal capital asset pricing model (ICAPM) shows that such assets should deliver lower average returns in equilibrium if they are priced from conservative long-term investors' first-order conditions.

Investment opportunities in the stock market may deteriorate either because expected stock returns decline or because the volatility of stock returns increases. The relative importance of these two types of intertemporal risk is an empirical question. In this paper, we estimate an econometric model of stock returns that captures time-variation in both expected returns and volatility and permits tractable analysis of long-term portfolio choice. The model is a vector autoregression (VAR) for aggregate stock returns, realized variance, and state variables, restricted to have scalar affine stochastic volatility so that the volatilities of all shocks move proportionally.

Using this model and the first-order conditions of an infinitely-lived investor with Epstein-Zin (1989, 1991) preferences, who is assumed to hold an aggregate stock index, we calculate the risk aversion needed to make the investor content to hold the market index rather than tilting his portfolio towards value stocks that offer higher average returns. We find that a moderate level of risk aversion, around 7, is sufficient to dissuade the investor from a tilt towards value stocks. Growth stocks are attractive to a moderately conservative long-term investor because they hedge against both declines in expected market returns and increases in market volatility. These considerations would not be relevant for a single-period investor.

We obtain similar results for several other equity portfolio tilts, including tilts to portfolios of stocks sorted by their past betas with market returns and with long-run volatility shocks, and to managed portfolios that vary equity exposure in response to the level of expected volatility. The major exception is that the conservative long-term investor would find it attractive to hold a managed portfolio that varies equity exposure in response to time-variation in expected stock returns. The reason is that we estimate only a weak correlation between expected returns and volatility, so a market timing strategy does not lead to an undesired volatility exposure.

Following Merton (1973), one might interpret the conservative long-term investor we consider in this paper as a representative investor who trades freely in all asset markets. There are however two obstacles to this interpretation. First, as already mentioned, our model does not explain why such an agent would not vary equity exposure with the level of the equity premium. Borrowing constraints can fix equity exposure at 100% when they bind, but we estimate that they will not bind at all times in our historical sample. Second, the aggregate stock index we consider here may not be an adequate proxy for all wealth, a point emphasized by many papers including Campbell (1996), Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Lustig, Van Nieuwerburgh, and Verdelhan (2013).

For both these reasons, we interpret our results in microeconomic terms, as a description of the intertemporal considerations that limit the desire of conservative long-term equity investors (including institutions such as pension funds and endowments) to follow value strategies and other equity strategies with high average returns. These considerations may contribute to the explanation of cross-sectional patterns in stock returns in a general equilibrium setting with heterogeneous investors, even if they do not provide a complete explanation in themselves.

Our empirical model provides a novel description of stochastic equity volatility that is of independent interest. Our VAR system includes not only stock returns and realized variance, but also other financial indicators including the price-smoothed earnings ratio and the default spread, the yield spread of low-rated over high-rated bonds. We find low-frequency movements in volatility tied to these variables. While this phenomenon has received little attention in the literature, we argue that it is a natural outcome of investor behavior. Investors in risky bonds perceive the long-run component of volatility and incorporate this information when they set credit spreads, as risky bonds are short the option to default over long maturities. GARCH-based methods that filter only the information in past stock returns fail to extract this low-frequency component of volatility, which is of key importance to long-horizon investors who care mostly about persistent changes in their investment opportunity set.

The organization of our paper is as follows. Section 2 reviews related literature. Section 3 presents the first-order conditions of an infinitely-lived Epstein-Zin investor, allowing for a specific form of stochastic volatility, and shows how they can be used to estimate preference parameters. Section 4 presents data, econometrics, and VAR estimates of the dynamic process for stock returns and realized volatility. This section documents the empirical success of our model in forecasting long-run volatility. Section 5 introduces our test assets and estimates their betas with news about the market's future cash flows, discount rates, and volatility. Section 6 turns to cross-sectional asset pricing and estimates the investor's preference parameters to fit a cross-section of excess returns on test assets, taking the dynamics of stock returns as given. This section also explores the implications of our model for the history of our investor's marginal utility. Section 7 concludes. An online appendix to the paper (Campbell, Giglio, Polk, and Turley 2015a) provides supporting details including a battery of robustness tests, and a companion paper (Campbell, Giglio, Polk, and Turley 2015b) considers non-equity test assets.

2 Literature Review

Since Merton (1973) first formulated the ICAPM, a large empirical literature has explored the relevance of intertemporal considerations for the pricing of financial assets in general, and the cross-sectional pricing of stocks in particular. One strand of this literature uses the approximate accounting identity of Campbell and Shiller (1988a) and the first-order conditions of an infinitely-lived investor with Epstein-Zin preferences to obtain approximate closed-form solutions for the ICAPM's risk prices (Campbell 1993). These solutions can be implemented empirically if they are combined with vector autoregressive (VAR) estimates of asset return dynamics. Campbell and Vuolteenaho (CV 2004), Campbell, Polk, and Vuolteenaho (2010), and Campbell, Giglio, and Polk (CGP 2013) use this approach to argue that value stocks outperform growth stocks on average because growth stocks hedge long-term investors against declines in the expected return on the aggregate stock market.

A weakness of these papers is that they ignore time-variation in the volatility of stock returns. We remedy this weakness by augmenting the VAR system with a scalar affine stochastic volatility model, in which a single state variable governs the volatility of all shocks to the VAR. In the continuous-time limit of the model, volatility always remains positive.[2] We extend the approximate closed-form ICAPM to allow for this type of stochastic volatility, and derive three priced risk factors corresponding to three important attributes of aggregate market returns: revisions in expected future cash flows, discount rates, and volatility.

An attractive feature of our model is that the prices of these three risk factors depend on only one free parameter, the long-horizon investor's coefficient of risk aversion. This protects our empirical analysis from the critique of Daniel and Titman (1997, 2012) and Lewellen, Nagel, and Shanken (2010) that models with multiple free parameters can spuriously fit the returns to a set of test assets with a low-order factor structure. Our use of risk-sorted test assets further protects us from this critique.

[2] Affine stochastic volatility models date back at least to Heston (1993) in continuous time. Similar models have been applied in the long-run risk literature by Eraker (2008) and Hansen (2012), among others, but much of this literature uses volatility specifications that are not guaranteed to remain positive.

Our work is complementary to recent research on the "long-run risk model" of asset prices (Bansal and Yaron 2004) which can be traced back to insights in Kandel and Stambaugh (1991). Both the approximate closed-form ICAPM and the long-run risk model start with the first-order conditions of an infinitely-lived Epstein-Zin investor. As originally stated by Epstein and Zin (1989), these first-order conditions involve both aggregate consumption growth and the return on the market portfolio of aggregate wealth. Campbell (1993) pointed out that the intertemporal budget constraint could be used to substitute out consumption growth, turning the model into a Merton-style ICAPM. Restoy and Weil (1998, 2011) used the same logic to substitute out the market portfolio return, turning the model into a generalized consumption CAPM in the style of Breeden (1979). Bansal and Yaron (2004) added stochastic volatility to the Restoy-Weil model, and subsequent theoretical and empirical research in the long-run risk framework has increasingly emphasized the importance of stochastic volatility (Bansal, Kiku, and Yaron 2012, Beeler and Campbell 2012, Hansen 2012). In this paper, we give the approximate closed-form ICAPM the same ability to handle stochastic volatility that its cousin, the long-run risk model, already possesses.[3]

Bansal, Kiku, Shaliastovich and Yaron (BKSY 2014), a paper written contemporaneously with the first version of this paper, explores the effects of stochastic volatility in the long-run risk model. Like us, they find stochastic volatility to be an important feature in the time series of equity returns. An important difference is that BKSY's benchmark model assumes a homoskedastic process driving volatility. In our theoretical analysis, we discuss some conditions that are required for their model solution to be valid and argue that these conditions are not satisfied empirically. The different modeling assumptions and some differences in empirical implementation account for our contrasting empirical results; we show that volatility risk is very important in explaining the cross-section of stock returns while they find it has little impact on cross-sectional differences in risk premia. Indeed, BKSY find that a value-minus-growth bet has a positive beta with volatility news, while we

[3]Two unpublished papers by Chen (2003) and Sohn (2010) also attempt to do this. As we discuss in detail in the online appendix, these papers make strong assumptions about the covariance structure of various news terms when deriving their pricing equations.

find it always has a negative volatility beta. Our negative volatility beta estimate is more consistent with models of real options held by growth firms, such as McQuade (2012), and with the underperformance of value stocks during periods of elevated volatility including the Great Depression, the technology boom of the late 1990s, and the Great Recession of the late 2000s (CGP 2013).

Stochastic volatility has been explored in other branches of the finance literature that we summarize in the online appendix. Most obviously, this is a prime concern of the field of financial econometrics. However, the focus has mostly been on univariate models, such as the GARCH class of models (Engle 1982, Bollerslev 1986), or univariate filtering methods that use realized high-frequency volatility (Barndorff-Nielsen and Shephard 2002, Andersen et al. 2003). A much smaller literature has, like us, looked directly at the information in other economic and financial variables concerning future volatility (Schwert 1989, Christiansen, Schmeling, and Schrimpf 2012, Paye 2012, Engle, Ghysels, and Sohn 2013).

3 An Intertemporal Model with Stochastic Volatility

In this section, we derive an expression for the log stochastic discount factor (SDF) of the intertemporal CAPM model that allows for stochastic volatility. We then discuss the properties of the model, including the requirements for a solution to exist, the implications for asset pricing, and methods for estimation.

3.1 The stochastic discount factor

3.1.1 Preferences

We assume a representative agent with Epstein–Zin preferences and write the value function as

$$V_t = \left[(1-\delta)\, C_t^{\frac{1-\gamma}{\theta}} + \delta \left(\mathrm{E}_t \left[V_{t+1}^{1-\gamma}\right]\right)^{1/\theta}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where C_t is consumption and the preference parameters are the discount factor δ, risk aversion γ, and the elasticity of intertemporal substitution (EIS) ψ. For convenience, we define $\theta = (1-\gamma)/(1-1/\psi)$.

The corresponding stochastic discount factor (SDF) can be written as

$$M_{t+1} = \left(\delta \left(\frac{C_t}{C_{t+1}}\right)^{1/\psi}\right)^{\theta} \left(\frac{W_t - C_t}{W_{t+1}}\right)^{1-\theta}, \tag{2}$$

where W_t is the market value of the consumption stream owned by the agent, including current consumption C_t.[4]

We will be studying risk premia and are therefore concerned with innovations in the SDF. We will also assume that asset returns and the SDF are conditionally jointly lognormally distributed. Since we allow for changing conditional moments, we are careful to write both first and second moments with time subscripts to indicate that they can vary over time. Defining the log return on wealth $r_{t+1} = \ln\left(W_{t+1}/\left(W_t - C_t\right)\right)$, and the log consumption-wealth ratio $h_{t+1} = \ln\left(W_{t+1}/C_{t+1}\right)$ (denoted by h because this is the variable that determines intertemporal hedging demand), we can write the innovation in the log SDF as

$$\begin{aligned} m_{t+1} - \mathrm{E}_t m_{t+1} &= -\frac{\theta}{\psi}(\Delta c_{t+1} - \mathrm{E}_t \Delta c_{t+1}) + (\theta - 1)\left(r_{t+1} - \mathrm{E}_t r_{t+1}\right) \\ &= \frac{\theta}{\psi}(h_{t+1} - \mathrm{E}_t h_{t+1}) - \gamma(r_{t+1} - \mathrm{E}_t r_{t+1}). \end{aligned} \tag{3}$$

The second equality uses the identity $r_{t+1} - \mathrm{E}_t r_{t+1} = (\Delta c_{t+1} - \mathrm{E}_t \Delta c_{t+1}) + (h_{t+1} - \mathrm{E}_t h_{t+1})$ to substitute consumption out of the SDF, replacing it with the wealth-consumption ratio and the log return on the wealth portfolio.

[4] This notational convention is not consistent in the literature. Some authors exclude current consumption from the definition of current wealth.

3.1.2 Solving the SDF forward

The online appendix shows that by using equation (3) to price the wealth portfolio, and taking a loglinear approximation of the wealth portfolio return (that is perfectly accurate when the elasticity of intertemporal substitution equals one), we obtain a difference equation for the innovation in h_{t+1} that can be solved forward to an infinite horizon to obtain:

$$\begin{aligned} h_{t+1} - \mathrm{E}_t h_{t+1} &= (\psi - 1)(\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} \\ &\quad + \frac{1}{2}\frac{\psi}{\theta}(\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j \mathrm{Var}_{t+j}\left[m_{t+1+j} + r_{t+1+j}\right] \\ &= (\psi - 1) N_{DR,t+1} + \frac{1}{2}\frac{\psi}{\theta} N_{RISK,t+1}, \end{aligned} \tag{4}$$

where ρ is a parameter of loglinearization related to the average consumption-wealth ratio, and somewhat less than one. The second equality in (4) follows CV (2004) and uses the notation N_{DR} ("news about discount rates") for revisions in expected future returns. In a similar spirit, we write revisions in expectations of future risk (the variance of the future log return plus the log stochastic discount factor) as N_{RISK}.

Substituting (4) into (3) and simplifying, we obtain:

$$\begin{aligned} m_{t+1} - \mathrm{E}_t m_{t+1} &= -\gamma\left[r_{t+1} - \mathrm{E}_t r_{t+1}\right] - (\gamma - 1) N_{DR,t+1} + \frac{1}{2} N_{RISK,t+1} \\ &= -\gamma N_{CF,t+1} - \left[-N_{DR,t+1}\right] + \frac{1}{2} N_{RISK,t+1}. \end{aligned} \tag{5}$$

The first equality in (5) expresses the log SDF in terms of the market return and news about future variables. In particular, it identifies three priced factors: the market return (with a price of risk γ), discount rate news (with price of risk $(\gamma - 1)$), and news about future risk (with price of risk of $-\frac{1}{2}$). This is an extension of the ICAPM as derived by Campbell (1993), with no reference to consumption or the elasticity of intertemporal substitution ψ.[5]

[5] Campbell (1993) briefly considers the heteroskedastic case, noting that when $\gamma = 1$, $\mathrm{Var}_t\left[m_{t+1} + r_{t+1}\right]$ is a constant. This implies that N_{RISK} does not vary over time so the stochastic volatility term disappears.

The second equality rewrites the model, following CV (2004), by breaking the market return into cash-flow news and discount-rate news. Cash-flow news $N_{CF,t+1}$ is defined by $N_{CF,t+1} = r_{t+1} - \mathrm{E}_t r_{t+1} + N_{DR,t+1}$. The price of risk for cash-flow news is γ times greater than the unit price of risk for (negative) discount-rate news, hence CV call betas with cash-flow news "bad betas" and those with negative discount-rate news "good betas". The third term in (5) shows the risk price for exposure to news about future risks and did not appear in CV's model, which assumed homoskedasticity. Not surprisingly, the model implies that an asset providing positive returns when risk expectations increase will offer a lower return on average; equivalently, the log SDF is high when future volatility is anticipated to be high.

Because the elasticity of intertemporal substitution (EIS) has no effect on risk prices in our model, we do not identify this parameter and therefore do not face the recent critique of Epstein, Farhi, and Strzalecki (2014) that models with a large wedge between risk aversion and the reciprocal of the EIS imply an unrealistic willingness to pay for early resolution of uncertainty.[6] However, the EIS does influence the implied behavior of the investor's consumption, a topic we explore further below.

3.1.3 From news about risk to news about volatility

The risk news term $N_{RISK,t+1}$ in equation (5) represents news about the conditional variance of returns plus the stochastic discount factor, $\mathrm{Var}_t\left[m_{t+1} + r_{t+1}\right]$. It therefore depends on the SDF and its innovations. To close the model and derive its empirical implications, we must make assumptions concerning the nature of the data generating process for stock returns and the variance terms that will allow us to solve for $\mathrm{Var}_t\left[m_{t+1} + r_{t+1}\right]$ and $N_{RISK,t+1}$.

Campbell claims that the stochastic volatility term also disappears when $\psi = 1$, but this is incorrect. When limits are taken correctly, N_{RISK} does not depend on ψ (except indirectly through the loglinearization parameter, ρ).

[6] We use the standard terminology to describe the two parameters of the Epstein-Zin utility function, γ as risk aversion and ψ as the elasticity of intertemporal substitution, although Garcia, Renault, and Semenov (2006) and Hansen, Heaton, Lee, and Roussanov (2007) point out that this interpretation may not be correct when γ differs from the reciprocal of ψ.

We assume that the economy is described by a first-order VAR

$$\mathbf{x}_{t+1} = \bar{\mathbf{x}} + \mathbf{\Gamma}\left(\mathbf{x}_t - \bar{\mathbf{x}}\right) + \sigma_t \mathbf{u}_{t+1}, \tag{6}$$

where $\mathbf{x}_{t+1}$ is an $n \times 1$ vector of state variables that has r_{t+1} as its first element, σ^2_{t+1} as its second element, and $n-2$ other variables that help to predict the first and second moments of aggregate returns. $\bar{\mathbf{x}}$ and $\mathbf{\Gamma}$ are an $n \times 1$ vector and an $n \times n$ matrix of constant parameters, and $\mathbf{u}_{t+1}$ is a vector of shocks to the state variables normalized so that its first element has unit variance. We assume that $\mathbf{u}_{t+1}$ has a constant variance-covariance matrix $\mathbf{\Sigma}$, with element $\Sigma_{11} = 1$. We also define $n \times 1$ vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, all of whose elements are zero except for a unit first element in $\mathbf{e}_1$ and second element in $\mathbf{e}_2$.

The key assumption here is that a scalar random variable, σ_t^2, equal to the conditional variance of market returns, also governs time-variation in the variance of all shocks to this system. Both market returns and state variables, including variance itself, have innovations whose variances move in proportion to one another. This assumption makes the stochastic volatility process affine, as in Heston (1993). It implies that the conditional variance of returns plus the stochastic discount factor is proportional to the conditional variance of returns themselves.

Given this structure, news about discount rates can be written as

$$N_{DR,t+1} = \mathbf{e}_1' \rho \mathbf{\Gamma}\left(\mathbf{I} - \rho\mathbf{\Gamma}\right)^{-1} \sigma_t \mathbf{u}_{t+1}, \tag{7}$$

while implied cash flow news is:

$$N_{CF,t+1} = \left(\mathbf{e}_1' + \mathbf{e}_1' \rho \mathbf{\Gamma}(\mathbf{I} - \rho\mathbf{\Gamma})^{-1}\right) \sigma_t \mathbf{u}_{t+1}. \tag{8}$$

Our log-linear model makes the log SDF a linear function of the state variables, so all shocks to the log SDF are proportional to σ_t, and $\mathrm{Var}_t\left[m_{t+1} + r_{t+1}\right] = \omega\sigma_t^2$ for some constant parameter ω. This implies that news about risk, N_{RISK}, is proportional to news about market return variance, N_V:[7]

$$N_{RISK,t+1} = \omega\rho\mathbf{e}_2' \left(\mathbf{I} - \rho\mathbf{\Gamma}\right)^{-1} \sigma_t \mathbf{u}_{t+1} = \omega N_{V,t+1}. \tag{9}$$

The parameter ω is a nonlinear function of the coefficient of relative risk aversion γ, as well as the VAR parameters and the loglinearization coefficient ρ, but it does not depend on the elasticity of intertemporal substitution ψ except indirectly through the influence of ψ on ρ. In the online appendix, we show that ω solves:

$$\omega\sigma_t^2 = (1-\gamma)^2 \mathrm{Var}_t \left[N_{CF,t+1}\right] + \omega(1-\gamma)\mathrm{Cov}_t \left[N_{CF,t+1}, N_{V,t+1}\right] + \omega^2 \frac{1}{4} \mathrm{Var}_t \left[N_{V,t+1}\right]. \tag{10}$$

There are two main channels through which γ affects ω. First, a higher risk aversion—given the underlying volatilities of all shocks—implies a more volatile stochastic discount factor m, and therefore higher risk. This effect is proportional to $(1-\gamma)^2$, so it increases rapidly with γ. Second, there is a feedback effect on current risk through future risk: ω appears on the right-hand side of the equation as well. Given that in our estimation we find $\mathrm{Cov}_t \left[N_{CF,t+1}, N_{V,t+1}\right] < 0$, this second effect makes ω increase even faster with γ.

The quadratic equation (10) has two solutions, but the online appendix shows that one of them can be disregarded. The false solution is easily identified by its implication that ω becomes infinite as volatility shocks become small. The appendix also shows how to write (10) directly in terms of the VAR parameters.

Finally, substituting (9) into (5), we obtain an empirically testable expression for the

[7]This property does not generally hold in the model with a homoskedastic process for σ_t^2 proposed by BKSY (2014). In BKSY's model, $\mathrm{Var}_t(m_{t+1} + r_{t+1})$ is not in general proportional to σ_t^2, but depends on both σ_t^2 and σ_t. Therefore, N_{RISK} is not in general proportional to N_V, and N_V is not in general the right news term to use in cross-sectional asset pricing. The online appendix shows that BKSY's use of N_V for asset pricing in this model can only be justified with additional special assumptions not stated by BKSY: that N_{CF} and N_V are uncorrelated, and that the N_V shock only depends on innovations of state variables which are themselves homoskedastic. As these assumptions are inconsistent with the data, and the homoskedastic process allows σ_t^2 to become negative even in continuous time, we do not further consider this model.

SDF innovations in the ICAPM with stochastic volatility:

$$m_{t+1} - \mathrm{E}_t m_{t+1} = -\gamma N_{CF,t+1} - [-N_{DR,t+1}] + \frac{1}{2}\omega N_{V,t+1}, \tag{11}$$

where ω solves equation (10).

3.2 Properties and estimation of the model

3.2.1 Existence of a solution

With constant volatility, our model can be solved for any level of risk aversion, but in the presence of stochastic volatility the model admits a solution only for values of risk aversion consistent with the existence of a real solution to the quadratic equation (10). Given our VAR estimates of the variance and covariance terms, the online appendix plots ω as a function of γ and shows that a real solution for ω exists when γ lies between zero and 7.2.

The online appendix also shows that existence of a real solution for ω requires γ to satisfy the upper bound:

$$\gamma \leq 1 - \frac{1}{(\rho_n - 1)\sigma_{cf}\sigma_v} \tag{12}$$

where ρ_n is the correlation between the news terms N_{CF} and N_V, σ_{cf} is the standard deviation of the scaled cash-flow news $N_{CF,t+1}/\sigma_t$, and σ_v is the standard deviation of the scaled variance news $N_{V,t+1}/\sigma_t$.

To develop the intuition behind these equations further, the online appendix studies a simple example in which the link between the existence to a solution for equation (10) and the existence of a value function for the representative agent can be shown analytically. The example assumes $\psi = 1$, since we can then solve directly for the value function without any need for a loglinear approximation of the return on the wealth portfolio (Tallarini 2000, Hansen, Heaton, and Li 2008). In the example, we find that the condition for the existence of the value function coincides precisely with the condition for the existence of a real solution to the quadratic equation for ω. This result shows that the possible non-existence of a solution to the quadratic equation for ω is a deep feature of the model, not an artifact of our loglinear approximation to the wealth portfolio return—which is not needed in the special case where $\psi = 1$. The problem arises because the value function becomes ever more sensitive to volatility as the volatility of the value function increases, and this sensitivity feeds back into the volatility of the value function, further increasing it. When this positive feedback becomes too powerful, then the value function ceases to exist.[8]

In our empirical analysis, we take seriously the constraint implied by the quadratic equation (10), and require that our parameter estimates satisfy this constraint.[9] Given the high average returns to risky assets in historical data, this means in practice that our estimate of risk aversion is often close to the estimated upper bound of 7.2.

3.2.2 Asset pricing equation and risk premia

To explore the implications of the model for risk premia, we use the general asset pricing equation under conditional lognormality,

$$0 = \ln \mathrm{E}_t \exp\{m_{t+1} + r_{i,t+1}\} = \mathrm{E}_t \left[m_{t+1} + r_{i,t+1}\right] + \frac{1}{2}\mathrm{Var}_t \left[m_{t+1} + r_{i,t+1}\right]. \tag{13}$$

Combining this with the approximation

$$\mathrm{E}_t r_{i,t+1} + \frac{1}{2}\sigma_{it}^2 \simeq \left(\mathrm{E}_t R_{i,t+1} - 1\right),$$

[8] In the online appendix, we show that existence of the solution for ω also imposes a lower bound on γ: $\gamma \geq 1 - (1/(\rho_n + 1)\sigma_{cf}\sigma_v)$. We do not focus on this lower bound on γ since in our case it lies far below zero, at -6.8.

[9] The constraint is ignored in BKSY (2014), when they consider the case of time-varying volatility of volatility as a robustness test in their Sections II.E and III.D. There, rather than imposing that ω and γ satisfy equation (10), they linearize the equation so that a solution exists for all values of γ, allowing combinations of (γ,ω) for which the true model does not have a solution. In the first draft of our paper we also used this inappropriate linearization.

which links expected log returns (adjusted by one-half their variance) to expected gross level returns $R_{i,t+1}$, and subtracting equation (13) for any reference asset j (which could be but does not need to be a true risk-free rate) from the equation for asset i, we can write a moment condition describing the relative risk premium of i relative to j as:

$$\begin{aligned} & \mathrm{E}_t \left[R_{i,t+1} - R_{j,t+1} + (r_{i,t+1} - r_{j,t+1})(m_{t+1} - \mathrm{E}_t m_{t+1}) \right] \\ = \; & \mathrm{E}_t \left[R_{i,t+1} - R_{j,t+1} - (r_{i,t+1} - r_{j,t+1})(\gamma N_{CF,t+1} + [-N_{DR,t+1}] - \frac{1}{2}\omega N_{V,t+1}) \right] = 0, \end{aligned} \quad (14)$$

where the second equality uses equation (11). This will be our main pricing equation: it contains all conditional implications of the model, for any pair of assets i and j. We note that in general the model does not restrict the covariances between the various assets' returns and the news terms: these are taken as given in the data and not derived from the theory (with the exception of the market portfolio itself, discussed in the next subsection).

We can alternatively write the moment conditions in covariance form:

$$\begin{aligned} \mathrm{E}_t \left[R_{i,t+1} - R_{j,t+1} \right] = & \; \gamma \mathrm{Cov}_t \left[r_{i,t+1} - r_{j,t+1}, N_{CF,t+1} \right] \\ & + \mathrm{Cov}_t \left[r_{i,t+1} - r_{j,t+1}, -N_{DR,t+1} \right] - \frac{1}{2}\omega \mathrm{Cov}_t \left[r_{i,t+1} - r_{j,t+1}, N_{V,t+1} \right]. \end{aligned} \quad (15)$$

As in CV (2004), this equation breaks an asset's overall covariance with unexpected returns on the wealth portfolio, $r_{t+1} - E_t r_{t+1} = N_{CF,t+1} - N_{DR,t+1}$, into two pieces, the first of which has a higher risk price than the second whenever $\gamma > 1$. Importantly, it also adds a third term capturing the asset's covariance with shocks to long-run expected future volatility.

3.2.3 Conditional and unconditional implications of the model

The moment condition (14) summarizes the conditional asset pricing implications of the model. It can be conditioned down to obtain the model's unconditional implications, replacing the conditional expectation in (14) with an unconditional expectation.

A special conditional implication of the model can be obtained when we focus on the wealth portfolio and the real risk free rate R_f. In this case, since both r_{t+1} and m_{t+1} are linear functions of the VAR state vector, their conditional covariance will be proportional to the stochastic variance term σ_t^2:

$$\mathrm{E}_t\left[R_{t+1} - R_{f,t+1}\right] = -\mathrm{Cov}_t\left[r_{t+1}, m_{t+1} - \mathrm{E}_t m_{t+1}\right] \propto \sigma_t^2 \tag{16}$$

The model implies that the risk premium on the market varies in proportion with the one-period conditional variance of the market.

This conditional restriction has some implications for the relation between news terms, in particular N_{DR} and N_V. While it does not tie the two terms precisely together (since N_{DR} also reflects news about the risk-free rate), it suggests that the two should be highly correlated: news about high future variance should correspond to news about high future discount rates. In the case that the risk-free rate is constant or at least unpredictable, the model predicts $N_{DR,t+1} \propto N_{V,t+1}$.

In our empirical implementation, we do not impose this restriction on the VAR. One reason for this is that we do not assume that we observe the riskless real return R^f_{t+1}. In addition, a large literature has shown that empirically this restriction fails to hold when standard empirical proxies for $R_{f,t+1}$ are used.[10] Consistent with this, we find that our empirical measure of σ_t^2, $EVAR$, does not significantly forecast returns in our unrestricted VAR.

However, in our empirical analysis we do test conditional asset pricing implications of the model by performing our GMM estimation using as instruments conditioning variables implied by the model (specifically σ_t^2). The only restriction we do not impose on the dynamics of returns in the VAR is the counterfactual tight link between N_{DR} and N_V.

[10] See for example Campbell (1987), Harvey (1989, 1991), or the review in Lettau and Ludvigson (2010).

3.2.4 Estimation

Estimation via GMM is straightforward in this model given the moment representation of the asset pricing equation (14). Conditional on the news terms, the model is a linear factor model (with the caveat that both level and log returns appear), which is easy to estimate via GMM even though it imposes nonlinear restrictions on the factor risk prices. The model has only one free parameter, γ, that determines the risk prices as γ for N_{CF}, 1 for $-N_{DR}$, and $-\omega(\gamma)/2$ for N_V, where $\omega(\gamma)$ is the solution of the quadratic equation (10) corresponding to γ and the estimated news terms.

We estimate the VAR parameters and the news terms separately via OLS, and use GMM to estimate the preference parameter γ. Our GMM standard errors for γ then condition on the estimated news terms. In theory, it would be possible to estimate both the dynamics and the moment conditions via GMM in one step. However, as discussed in CGP (2013), this estimation is numerically involved and unstable given the large number of parameters.

The moment condition (14) holds for any two assets i and j. If a real risk-free rate were available (which we would refer to as R_f), it would be a natural choice for the reference asset j. In our empirical analysis, we use a nominal Treasury bill as the reference asset, writing its return as R_{Tbill}, but we do not assume that this is a real riskless return. We do allow for a free intercept relative to the Treasury bill return, as in Black's (1972) implementation of the CAPM, to account for the fact that investors are unable to borrow at the Treasury bill rate but must typically pay a spread over this rate. The Treasury bill rate plus that spread is the zero-beta rate of the intertemporal model.

Finally, we perform our GMM estimation using a prespecified diagonal weighting matrix W whose elements are the inverse of the variances of the test assets. This approach ensures that the GMM estimation is not focusing on some extreme linear combination of the assets, while still taking into account the different variances of individual moment conditions. We have repeated our analysis using one-step and two-step efficient estimation, and the qualitative results in the paper continue to hold in these cases.

3.2.5 Implied consumption innovations

As in Campbell (1993), we can estimate the model without having to observe the consumption process of the investor. However, it is interesting to look at the model-implied consumption, and compare it with the observed process of aggregate consumption.

Consumption innovations for our investor are given by

$$\Delta c_{t+1} - E_t \Delta c_{t+1} = (r_{t+1} - E_t r_{t+1}) - (\psi - 1) N_{DR,t+1} - (\psi - 1)\frac{1}{2}\frac{\omega}{1-\gamma} N_{V,t+1}. \tag{17}$$

The EIS parameter ψ, which enters this equation, is not pinned down by our VAR estimation so we calibrate it to three different values, 0.5, 1.0, and 1.5. The online appendix shows that implied consumption volatility is positively related to ψ, given our VAR estimates of return dynamics. With $\psi = 0.5$, our investor's consumption (which need not equal aggregate consumption) is considerably more volatile than aggregate consumption but roughly as volatile as equity dividend growth. Implied and actual consumption growth are positively correlated, and more so when both series are exponentially smoothed.[11]

4 Predicting Aggregate Stock Returns and Volatility

4.1 State variables

Our full VAR specification of the vector $\mathbf{x}_{t+1}$ includes six state variables, four of which are among the five variables in CGP (2013). To those four variables, we add the Treasury bill

[11] An interesting exercise would be to confront our implied consumption series with microeconomic data on stockholders' consumption, as in Malloy, Moskowitz, and Vissing-Jørgensen (2009). However, the short sample period over which such data are available is an obstacle to this approach.

rate R_{Tbill} (using it instead of the term yield spread used by CGP) and an estimate of conditional volatility. The data are all quarterly, from 1926:2 to 2011:4.

The first variable in the VAR is the log real return on the market, r_M, the difference between the log return on the Center for Research in Securities Prices (CRSP) value-weighted stock index and the log return on the Consumer Price Index. This is a standard proxy for the aggregate wealth portfolio, but in the online appendix we consider alternative proxies that delever the market return by combining it in various proportions with Treasury bills.

The second variable is expected market variance ($EVAR$). This variable is meant to capture the variance of market returns, σ_t^2, conditional on information available at time t, so that innovations to this variable can be mapped to the N_V term described above. To construct $EVAR_t$, we proceed as follows. We first construct a series of within-quarter *realized* variance of daily returns for each time t, $RVAR_t$. We then run a regression of $RVAR_{t+1}$ on lagged realized variance ($RVAR_t$) as well as the other five state variables at time t. This regression then generates a series of predicted values for $RVAR$ at each time $t+1$, that depend on information available at time t: $\widehat{RVAR}_{t+1}$. Finally, we define our expected variance at time t to be exactly this predicted value at $t+1$:

$$EVAR_t \equiv \widehat{RVAR}_{t+1}.$$

Note that though we describe our methodology in a two-step fashion where we first estimate $EVAR$ and then use $EVAR$ in a VAR, this is only for interpretability. Indeed, this approach to modeling $EVAR$ can be considered a simple renormalization of equivalent results we would find from a VAR that included $RVAR$ directly.[12]

The third variable is the log of the S&P 500 price-smoothed earnings ratio (PE) adapted from Campbell and Shiller (1988b), where earnings are smoothed over ten years. The variable

[12] Since we weight observations based on $RVAR$ in the first stage and then reweight observations using $EVAR$ in the second stage, our two-stage approach in practice is not exactly the same as a one-stage approach. However, the online appendix shows that results from a $RVAR$-weighted, single-step estimation are qualitatively very similar to those produced by our two-stage approach.

is constructed as in GCP (2013). The fourth is the yield on a three-month Treasury Bill (R_{Tbill}) from CRSP. The fifth is the small-stock value spread (VS), constructed as described in CGP (2013).

The sixth and final variable is the default spread (DEF), defined as the difference between the log yield on Moody's BAA and AAA bonds, obtained from the Federal Reserve Bank of St. Louis. We include the default spread in part because that variable is known to track time-series variation in expected real returns on the market portfolio (Fama and French 1989), but also because shocks to the default spread should to some degree reflect news about aggregate default probabilities, which in turn should reflect news about the market's future cash flows and volatility.

4.2 Short-run volatility estimation

In order for the regression model that generates $EVAR_t$ to be consistent with a reasonable data-generating process for market variance, we deviate from standard OLS in two ways. First, we constrain the regression coefficients to produce fitted values (i.e. expected market return variance) that are positive. Second, given that we explicitly consider heteroskedasticity of the innovations to our variables, we estimate this regression using Weighted Least Squares (WLS), where the weight of each observation pair ($RVAR_{t+1}$, $\mathbf{x}_t$) is initially based on the time-t value of $(RVAR)^{-1}$. However, to ensure that the ratio of weights across observations is not extreme, we shrink these initial weights towards equal weights. In particular, we set our shrinkage factor large enough so that the ratio of the largest observation weight to the smallest observation weight is always less than or equal to five. Though admittedly somewhat ad hoc, this bound is consistent with reasonable priors on the degree of variation over time in the expected variance of market returns. More importantly, we show in the online appendix that our results are robust to variation in this bound. Both the constraint on the regression's fitted values and the constraint on WLS observation weights bind in the sample we study.

The first-stage regression generating the state variable $EVAR_t$ is reported in Table 1, Panel A. Perhaps not surprisingly, past realized variance strongly predicts future realized variance. More importantly, the regression documents that an increase in either PE or DEF predicts higher future realized volatility. Both of these results are strongly statistically significant and are a novel finding of the paper. The predictive power of very persistent variables like PE and DEF indicates a potentially important role for lower-frequency movements in stochastic volatility.

We argue that these empirical patterns are sensible. Investors in risky bonds incorporate their expectation of future volatility when they set credit spreads, as risky bonds are short the option to default. Therefore we expect higher DEF to predict higher $RVAR$. The positive predictive relationship between PE and $RVAR$ might seem surprising at first, but one has to remember that the coefficient indicates the effect of a change in PE holding constant the other variables, in particular the default spread DEF. Since the default spread should also generally depend on the equity premium and since most of the variation in PE is due to variation in the equity premium, we can regard PE as purging DEF of its equity premium component to reveal more clearly its forecast of future volatility. We discuss this interpretation further in section 4.4 below.

The R^2 of the variance forecasting regression is nearly 38%. The relatively low R^2 masks the fact that the fit is indeed quite good, as we can see from Figure 1, in which $RVAR$ and $EVAR$ are plotted together. The R^2 is heavily influenced by occasional spikes in realized variance, which the simple linear model we use is not able to capture. Indeed, our WLS approach downweights the importance of these spikes in the estimation procedure.

4.3 Estimation of the VAR and the news terms

4.3.1 VAR estimates

We estimate a first-order VAR as in equation (6), where $\mathbf{x}_{t+1}$ is a 6×1 vector of state variables ordered as follows:

$$\mathbf{x}_{t+1} = [r_{M,t+1}\ EVAR_{t+1}\ PE_{t+1}\ R_{Tbill,t+1}\ DEF_{t+1}\ VS_{t+1}]$$

so that the real market return $r_{M,t+1}$ is the first element and $EVAR$ is the second element. $\bar{\mathbf{x}}$ is a 6×1 vector of the means of the variables, and $\mathbf{\Gamma}$ is a 6×6 matrix of constant parameters. Finally, $\sigma_t \mathbf{u}_{t+1}$ is a 6×1 vector of innovations, with the conditional variance-covariance matrix of $\mathbf{u}_{t+1}$ a constant $\mathbf{\Sigma}$, so that the parameter σ_t^2 scales the entire variance-covariance matrix of the vector of innovations.

The first-stage regression forecasting realized market return variance described in the previous section generates the variable $EVAR$. The theory in Section 3 assumes that σ_t^2, proxied for by $EVAR$, scales the variance-covariance matrix of state variable shocks. Thus, as in the first stage, we estimate the second-stage VAR using WLS, where the weight of each observation pair $(\mathbf{x}_{t+1}, \mathbf{x}_t)$ is initially based on $(EVAR_t)^{-1}$. We continue to constrain both the weights across observations and the fitted values of the regression forecasting $EVAR$.

Table 1, Panel B presents the results of the VAR estimation for the full sample (1926:2 to 2011:4). We report bootstrap standard errors for the parameter estimates of the VAR that take into account the uncertainty generated by forecasting variance in the first stage. Consistent with previous research, we find that PE negatively predicts future returns, though the t-statistic indicates only marginal significance. The value spread has a negative but not statistically significant effect on future returns. In our specification, a higher conditional variance, $EVAR$, is associated with higher future returns, though the effect is not statistically significant. Of course, the relatively high degree of correlation among PE, DEF, VS, and

$EVAR$ complicates the interpretation of the individual effects of those variables. As for the other novel aspects of the transition matrix, both high PE and high DEF predict higher future conditional variance of returns. High past market returns forecast lower $EVAR$, higher PE, and lower DEF.[13]

Table 1, Panel C reports the sample correlation matrices of both the unscaled residuals $\sigma_t \mathbf{u}_{t+1}$ and the scaled residuals $\mathbf{u}_{t+1}$. The correlation matrices report standard deviations on the diagonals. A comparison of the standard deviations of the unscaled and scaled residuals provides a rough indication of the effectiveness of our empirical solution to the heteroskedasticity of the VAR. In general, the standard deviations of the scaled residuals are several times larger than their unscaled counterparts. More specifically, our approach implies that the scaled return residuals should have unit standard deviation. Our implementation results in a sample standard deviation of 1.14.[14]

Table 1, Panel D reports the coefficients of a regression of the squared unscaled residuals $\sigma_t u_{t+1}$ of each VAR equation on a constant and $EVAR$. These results are broadly consistent with our assumption that $EVAR$ captures the conditional volatility of the market return and other state variables. The coefficient on $EVAR$ in the regression forecasting the squared market return residuals is 1.85, rather than the theoretically expected value of one, but this coefficient is sensitive to the weighting scheme used in the regression. The fact that $EVAR$ significantly predicts with a positive sign all the squared errors of the VAR shows that the volatilities of all innovations are driven by a common factor, as we assume, although of course it is possible that empirically, other factors also influence the volatilities of certain variables.

[13] One worry is that many of the elements of the transition matrix are estimated imprecisely. Though these estimates may be zero, their non-zero but statistically insignificant in-sample point estimates, in conjunction with the highly-nonlinear function that generates discount-rate and volatility news, may result in misleading estimates of risk prices. However, the online appendix shows that results are qualitatively similar if we instead employ a partial VAR where, via a standard iterative process, only variables with t-statistics greater than 1.0 are included in each VAR regression.

[14] A comparison of the unscaled and scaled autocorrelation matrices, in the online appendix, reveals in addition that much of the sample autocorrelation in the unscaled residuals is eliminated by our WLS approach.

4.3.2 News terms

The top panel of Table 2 presents the variance-covariance matrix and the standard deviation/correlation matrix of the news terms, estimated as described above. Consistent with previous research, we find that discount-rate news is nearly twice as volatile as cash-flow news.

The interesting new results in this table concern the variance news term N_V. First, news about future variance has significant volatility, with nearly a third of the variability of discount-rate news. Second, variance news is negatively correlated (-0.12) with cash-flow news: as one might expect from the literature on the "leverage effect" (Black 1976, Christie 1982), news about low cash flows is associated with news about higher future volatility. This finding makes it unappealing to assume that variance news and cash-flow news are uncorrelated, as would be required for the validity of the model solution in BKSY (2014). Third, N_V is close to uncorrelated (-0.03) with discount-rate news.[15] The net effect of these correlations, documented in the lower left panel of Table 2, is a slightly negative correlation of -0.03 between our measure of volatility news and contemporaneous market returns.

The lower right panel of Table 2 reports the decomposition of the vector of innovations $\sigma_t^2 u_{t+1}$ into the three terms $N_{CF,t+1}, N_{DR,t+1}$, and $N_{V,t+1}$. As shocks to $EVAR$ are just a linear combination of shocks to the underlying state variables, which includes $RVAR$, we "unpack" $EVAR$ to express the news terms as a function of r_M, PE, R_{Tbill}, VS, DEF, and $RVAR$. The panel shows that innovations to $RVAR$ are mapped more than one-to-one to news about future volatility. However, several of the other state variables also drive news about volatility. Specifically, we find that innovations in PE, DEF, and VS are associated with news of higher future volatility. This panel also indicates that all state variables with the exception of R^{Tbill} are statistically significant in terms of their contribution to at least one of the three news terms. We choose to leave R_{Tbill} in the VAR, though its presence in

[15] Though the point estimate of this correlation is negative, the large standard error implies that we cannot reject the "volatility feedback effect" (Campbell and Hentschel 1992, Calvet and Fisher 2007), which generates a positive correlation. For related research see French, Schwert, and Stambaugh (1987).

the system makes little difference to our conclusions.

Figure 2 plots the smoothed series for N_{CF}, $-N_{DR}$ and N_V using an exponentially-weighted moving average with a quarterly decay parameter of 0.08. This decay parameter implies a half-life of approximately two years. The pattern of N_{CF} and $-N_{DR}$ we find is consistent with previous research. As a consequence, we focus on the smoothed series for market variance news. There is considerable time variation in N_V, and in particular we find episodes of news of high future volatility during the Great Depression and just before the beginning of World War II, followed by a period of little news until the late 1960s. From then on, periods of positive volatility news alternate with periods of negative volatility news in cycles of 3 to 5 years. Spikes in news about future volatility are found in the early 1970s (following the oil shocks), in the late 1970s and again following the 1987 crash of the stock market. The late 1990s are characterized by strongly negative news about future returns, and at the same time higher expected future volatility. The recession of the late 2000s is instead characterized by strongly negative cash-flow news, together with a spike in volatility of the highest magnitude in our sample. The recovery from the financial crisis has brought positive cash-flow news together with news about lower future volatility.

4.4 Predicting long-run volatility

The predictability of volatility, and especially of its long-run component, is central to this paper. In the previous sections, we have shown that volatility is strongly predictable, specifically by variables beyond lagged realizations of volatility itself: PE and DEF contain essential information about future volatility. We have also proposed a VAR-based methodology to construct long-horizon forecasts of volatility that incorporate all the information in lagged volatility as well as in the additional predictors like PE and DEF.

We now ask how well our proposed long-run volatility forecast captures the long-horizon component of volatility. In the online appendix we regress realized, discounted, annualized long-run variance up to period h,

$$LHRVAR_h = \frac{4\Sigma_{j=1}^{h} \rho^{j-1} RVAR_{t+j}}{\Sigma_{j=1}^{h} \rho^{j-1}},$$

on the variables included in our VAR system, the VAR long-horizon forecast, and some alternative forecasts of long-run variance. We focus on a 10-year horizon ($h = 40$) as longer horizons come at the cost of fewer independent observations; however, the online appendix confirms that our results are robust to horizons ranging from one to 15 years.

As alternatives to the VAR approach, we estimate two standard GARCH-type models, specifically designed to capture the long-run component of volatility: the two-component exponential (EGARCH) model proposed by Adrian and Rosenberg (2008), and the fractionally integrated (FIGARCH) model of Baillie, Bollerslev, and Mikkelsen (1996). We first estimate both GARCH models using the full sample of daily returns and then generate the appropriate forecast of $LHRVAR_{40}$. To these two models, we add the set of variables from our VAR, and compare the forecasting ability of these different models. We find that while the EGARCH and FIGARCH forecasts do forecast long-run volatility, our VAR variables provide as good or better explanatory power, and $RVAR$, PE and DEF are strongly statistically significant. Our long-run VAR forecast has a coefficient of 1.02, which remains highly significant at 0.82 even in the presence of the FIGARCH forecast. We also find that DEF does not predict long-horizon volatility in the presence of our VAR forecast, implying that the VAR model captures the long-horizon information in the default spread.

The online appendix also examines more carefully the links between PE, DEF, and $LHRVAR_{40}$. We find that by itself, PE has almost no information about low-frequency variation in volatility. In contrast, DEF forecasts nearly 22% of the variation in $LHRVAR_{40}$. And if we use the component of DEF that is orthogonal to PE, which we call $DEFO$, the R^2 increases to over 51%. Our interpretation of these results is that DEF contains information about future volatility because risky bonds are short the option to default. However, DEF also contains information about future aggregate risk premia. We know from previous work that most of the variation in PE reflects aggregate risk premia. Therefore, including PE in the volatility forecasting regression cleans up variation in DEF due to aggregate risk premia and thus sharpens the link between DEF and future volatility. Since PE and DEF are negatively correlated (default spreads are relatively low when the market trades rich), both PE and DEF receive positive coefficients in the multiple regression.

Figure 3 provides a visual summary of the long-run volatility-forecasting power of our key VAR state variables and our interpretation. The top panel plots $LHRVAR_{40}$ together with lagged DEF and PE. The graph confirms the strong negative correlation between PE and DEF (correlation of -0.6) and highlights how both variables track long-run movements in long run volatility. To isolate the contribution of the default spread in predicting long run volatility, the bottom panel plots $LHRVAR_{40}$ together with $DEFO$. The improvement in fit moving from the top panel to the bottom panel is clear.

The contrasting behavior of DEF and $DEFO$ in the two panels during episodes such as the tech boom help illustrate the workings of our story. Taken in isolation, the relatively stable default spread throughout most of the late 1990s would predict little change in future market volatility. However, once the declining equity premium over that period is taken into account (as shown by the rapid increase in PE), one recognizes that a PE-adjusted default spread in the late 1990s actually forecasted much higher volatility ahead.

As a further check on the usefulness of our VAR approach, in the online appendix we compare our variance forecasts to option-implied variance forecasts over the period 1998–2011. We find that when both the VAR and option data are used to predict realized variance, the VAR forecasts drive out the option-implied forecasts while remaining statistically and economically significant.

Taken together, these results make a strong case that credit spreads and valuation ratios contain information about future volatility not captured by simple univariate models, even those designed to fit long-run movements in volatility, and that our VAR method for calculating long-horizon forecasts preserves this information.

5 Test Assets and Beta Measurement

5.1 Test assets

In addition to the six VAR state variables, our analysis requires excess returns on a cross section of test assets. We construct several sets of portfolios for this purpose, reporting details on the construction method in the online appendix.

Our primary cross section consists of the excess returns over Treasury bills on the 25 ME- and BE/ME-sorted portfolios, studied in Fama and French (1993), extended in Davis, Fama, and French (2000), and made available by Professor Kenneth French on his website. We consider two main subsamples: early (1931:3-1963:3) and modern (1963:4-2011:4) due to the findings in CV (2004) of important differences in the risks of these portfolios between the early and modern period.

To guard against the concerns of Daniel and Titman (1997, 2012) and Lewellen, Nagel, and Shanken (2010) that characteristic-sorted portfolios may have a low-order factor structure that is easily fit by spurious models, we construct a second set of six portfolios double-sorted on past multiple betas with market returns and variance innovations (approximated by a weighted average of changes in the VAR explanatory variables).

We also consider excess returns on equity portfolios that are formed based on both characteristics and past risk loadings. One possible explanation for our finding that growth stocks hedge volatility relative to value stocks is that growth firms are more likely to hold real options, whose value increases with volatility. To test this interpretation, we first sort stocks based on two firm characteristics that are often used to proxy for the presence of real options and that are available for a large percentage of firms throughout our sample period: BE/ME and idiosyncratic volatility (*ivol*). Having formed nine portfolios using a two-way characteristic sort, we split each of these portfolios into two subsets based on pre-formation estimates of each stock's simple beta with variance innovations. One might expect that sorts on simple rather than partial betas will be more effective in establishing a link between pre-formation and post-formation estimates of volatility beta, since the market is correlated with volatility news.

5.2 Beta measurement

We first examine the betas implied by the covariance form of the model in equation (15). We cosmetically multiply and divide all three covariances by the sample variance of the unexpected log real return on the market portfolio to facilitate comparison to previous research, defining

$$\begin{aligned}
\beta_{i,CF_M} &\equiv \frac{Cov(r_{i,t} - r_{Tbill,t}, N_{CF,t})}{Var(r_{M,t} - E_{t-1}r_{M,t})}, \\
\beta_{i,DR_M} &\equiv \frac{Cov(r_{i,t} - r_{Tbill,t}, -N_{DR,t})}{Var(r_{M,t} - E_{t-1}r_{M,t})}, \\
\text{and } \beta_{i,V_M} &\equiv \frac{Cov(r_{i,t} - r_{Tbill,t}, N_{V,t})}{Var(r_{M,t} - E_{t-1}r_{M,t})}.
\end{aligned}$$

The risk prices on these betas are just the variance of the market return innovation times the risk prices in equation (15).

We estimate cash-flow, discount-rate, and variance betas using the fitted values of the market's cash flow, discount-rate, and variance news estimated in the previous section. Specifically, we estimate simple WLS regressions of each portfolio's log returns on each news term, weighting each time-$t+1$ observation pair by the weights used to estimate the VAR in Table 1 Panel B. We then scale the regression loadings by the ratio of the sample variance of the news term in question to the sample variance of the unexpected log real return on the market portfolio to generate estimates for our three-beta model.

5.2.1 Characteristic-sorted portfolios

Table 3 Panel A shows the estimated betas for the 25 size- and book-to-market portfolios over the 1931-1963 period. The portfolios are organized in a square matrix with growth stocks at the left, value stocks at the right, small stocks at the top, and large stocks at the bottom. At the right edge of the matrix we report the differences between the extreme growth and extreme value portfolios in each size group; along the bottom of the matrix we report the differences between the extreme small and extreme large portfolios in each BE/ME category. The top matrix displays post-formation cash-flow betas, the middle matrix displays post-formation discount-rate betas, while the bottom matrix displays post-formation variance betas. In square brackets after each beta estimate we report a standard error, calculated conditional on the realizations of the news series from the aggregate VAR model.

In the pre-1963 sample period, value stocks have both higher cash-flow and higher discount-rate betas than growth stocks. An equal-weighted average of the extreme value stocks across size quintiles has a cash-flow beta 0.12 higher than an equal-weighted average of the extreme growth stocks. The difference in estimated discount-rate betas, 0.25, is in the same direction. Similar to value stocks, small stocks have higher cash-flow betas and discount-rate betas than large stocks in this sample (by 0.16 and 0.36, respectively, for an equal-weighted average of the smallest stocks across value quintiles relative to an equal-weighted average of the largest stocks). These differences are extremely similar to those in CV (2004), despite the exclusion of the 1929-1931 subperiod, the replacement of the excess log market return with the log real return, and the use of a richer, heteroskedastic VAR.

The new finding in the top portion of Table 3 Panel A is that value stocks and small stocks are also riskier in terms of volatility betas. An equal-weighted average of the extreme value stocks across size quintiles has a volatility beta 0.06 lower than an equal-weighted average of the extreme growth stocks. Similarly, an equal-weighted average of the smallest stocks across value quintiles has a volatility beta that is 0.06 lower than an equal-weighted average of the largest stocks. In summary, value and small stocks were unambiguously riskier than growth and large stocks over the 1931-1963 period.

Table 3 Panel B reports the corresponding estimates for the post-1963 period. As documented in this subsample by CV (2004), value stocks still have slightly higher cash-flow betas than growth stocks, but much lower discount-rate betas. Our new finding here is that value stocks continue to have much lower volatility betas, and the spread in volatility betas is even greater than in the early period. The volatility beta for the equal-weighted average of the extreme value stocks across size quintiles is 0.11 lower than the volatility beta of an equal-weighted average of the extreme growth stocks, a difference that is more than 85% higher than the corresponding difference in the early period.[16]

These results imply that in the post-1963 period where the CAPM has difficulty explaining the low returns on growth stocks relative to value stocks, growth stocks are relative hedges for two key aspects of the investment opportunity set. Consistent with CV (2004), growth stocks hedge news about future real stock returns. The novel finding of this paper is that growth stocks also hedge news about the variance of the market return.

One interesting aspect of these findings is the fact that the average β_V of the 25 size- and book-to-market portfolios changes sign from the early to the modern subperiod. Over the 1931-1963 period, the average β_V is -0.10 while over the 1964-2011 period this average becomes 0.06. Of course, given the strong positive link between PE and volatility news documented in the lower right panel of Table 2, one should not be surprised that the market's β_V can be positive. Nevertheless, in the online appendix we study this change in sign more carefully. We show that the market's beta with realized volatility has remained negative in the modern period, highlighting the important distinction between realized and expected future volatility. We also show that the change in the sign of β_V is driven by a change in the correlation between the aggregate market and $DEFO$, our simple proxy for news about long-horizon variance.

[16] Our findings are in sharp contrast to BKSY (2014), who estimate that value-minus-growth portfolios are volatility hedges. (See their Tables VII, X, and XI.) Their finding is hard to reconcile with theory (real option models such as McQuade 2012) and stylized facts (notably the performance of value-minus-growth bets during the Great Depression, the Tech Boom, and the Great Recession).

5.2.2 Risk-sorted portfolios

Panels C and D of Table 3 show the estimated betas for the six risk-sorted portfolios over the 1931-1963 and post-1963 periods. The portfolios are organized in a rectangular matrix with low market-beta stocks at the left, high market-beta stocks at the right, low volatility-beta stocks at the top, and high volatility-beta stocks at the bottom. Otherwise the format is the same as that of Panels A and B.

In the pre-1963 sample period, high market-beta stocks have both higher cash-flow and higher discount-rate betas than low market-beta stocks. Similarly, low volatility-beta stocks have higher cash-flow betas and discount-rate betas than high volatility-beta stocks. High market-beta stocks also have lower volatility betas, but sorting stocks by their past volatility betas induces little spread in post-formation volatility betas. Putting these results together, in the 1931-1963 period high market-beta stocks and low volatility-beta stocks were unambiguously riskier than low market-beta and high volatility-beta stocks.

In the post-1963 (modern) period, high market-beta stocks again have higher cash-flow and higher discount-rate betas than low market-beta stocks. However, high market-beta stocks now have higher volatility betas and are therefore safer in this dimension. Thus our three-beta model with priced volatility risk can potentially explain the well-known result that stocks with high past market betas have offered relatively little extra return in the past 50 years (Fama and French 1992).

In the modern period, sorts on volatility beta generate an economically and statistically significant spread in post-formation volatility beta. These high volatility-beta portfolios also tend to have higher discount-rate betas and lower cash-flow betas, though the patterns are not uniform.

We also examine test assets that are formed based on both characteristics and risk estimates. The online appendix reports the estimated betas for the 18 BE/ME-$ivol$-$\widehat{\beta}_{\Delta VAR}$-sorted portfolios in both the early and modern sample periods. In the early period, firms with higher *ivol* have lower post-formation volatility betas regardless of their book-to-market ratio. Consistent with this finding, higher *ivol* stocks have higher average returns. In the modern period, however, we find that among stocks with low BE/ME, firms with higher *ivol* have higher post-formation volatility betas and lower average returns; but these patterns reverse among stocks with high BE/ME.

We argue that these differences make economic sense. High idiosyncratic volatility increases the value of growth options, which is an important effect for growing firms with flexible real investment opportunities, but much less so for stable, mature firms. Valuable growth options in turn imply high betas with aggregate volatility shocks. Hence high idiosyncratic volatility naturally raises the volatility beta for growth stocks more than for value stocks. This effect is stronger in the modern sample where growing firms with flexible investment opportunities are more prevalent.

These results have the potential to explain the puzzling finding that high idiosyncratic-volatility stocks have lower average returns than low idiosyncratic-volatility stocks (Ang, Hodrick, Xing, and Zhang 2006 AHXZ), as well as the fact that the unconditional *ivol* effect is non-monotonic (AHXZ Table VI).[17] They may also explain why the *ivol* effect appears to be less robust in some samples using different methodologies (Bali and Cakici 2008) and even switches sign in others (Fu 2009), because different samples and weighting schemes may alter the value characteristic and hence the volatility beta of stocks with high idiosyncratic volatility.

Taken together, the findings from the characteristic- and risk-sorted test assets suggest that volatility betas vary with multiple stock characteristics, and that techniques that take this into account may be more effective in generating a spread in post-formation volatility beta.

[17] Barinov (2013) and Chen and Petkova (2014) also argue that the idiosyncratic volatility effect can be explained by aggregate volatility risk, but they do not use a theoretically-motivated volatility risk factor.

6 Pricing the Cross-Section of Stock Returns

We now turn to pricing the cross section of excess returns on our test assets. We estimate our model's single parameter via GMM, using the moment condition (14). For ease of exposition, we report our results in terms of the expected return-beta representation from equation (15), rescaled by the variance of market return innovations as in section 5.2:

$$\overline{R}_i - \overline{R}_{Tbill} = g_0 + g_1 \widehat{\beta}_{i,CF_M} + g_2 \widehat{\beta}_{i,DR_M} + g_3 \widehat{\beta}_{i,V_M} + e_i, \tag{18}$$

where bars denote time-series means and betas are measured using excess returns over Treasury bills.

We evaluate the performance of five asset-pricing models, all estimated via GMM: 1) the traditional CAPM that restricts cash-flow and discount-rate betas to have the same price of risk and sets the price of variance risk to zero; 2) the two-beta intertemporal asset pricing model of CV (2004) that restricts the price of discount-rate risk to equal the variance of the market return and again sets the price of variance risk to zero; 3) our three-beta intertemporal asset pricing model that restricts the price of discount-rate risk to equal the variance of the market return and constrains the prices of cash-flow and variance risk to be related by equation (10), with $\rho = 0.95$ per year; 4) a partially-constrained three-beta model that restricts the price of discount-rate risk to equal the variance of the market return but freely estimates the other two risk prices (effectively decoupling γ and ω); and 5) an unrestricted three-beta model that allows free risk prices for cash-flow, discount-rate, and volatility betas.

Each model is estimated in two different forms: one with a restricted zero-beta rate equal to the Treasury-bill rate as in the Sharpe-Lintner version of the CAPM, and one with an unrestricted zero-beta rate following Black (1972). Allowing for an unrestricted zero-beta rate may be particularly important given the extensive evidence in Krishnamurthy and Vissing-Jørgensen (2012) that Treasury Bills provide convenience benefits in terms of liquidity and safety.[18]

We present our main pricing results in the next two subsections. The online appendix examines the robustness of our results to a wide variety of methodological changes. This analysis includes using various subsets of variables in our baseline VAR, estimating the VAR in different ways, using different estimates of realized variance, altering the set of variables in the VAR, exploring the VAR's out-of-sample properties, using different proxies for the wealth portfolio including delevered equity portfolios, and varying both ρ and the excess zero-beta rate. Such robustness analysis is important because the VAR's news decomposition can be sensitive to the forecasting variables included.[19] We also use our model to understand Fama and French's (1993) risk factors, decomposing the volatility betas of both $RMRF$ and HML and linking the returns on HML to our three news terms.

6.1 Characteristic-sorted test assets

Table 4 reports separate results for the early sample period 1931-1963 (Panel A) and the modern sample period 1963-2011 (Panel B), using 25 size- and book-to-market-sorted portfolios as test assets. The table has ten columns, two specifications for each of our five asset pricing models. The first 8 rows of each panel in Table 4 are divided into four sets of two rows. The first set of two rows corresponds to the zero-beta rate (in excess of the Treasury-bill rate), the second set to the premium on cash-flow beta, the third set to the premium on discount-rate beta, and the fourth set to the premium on volatility beta. Within each set, the first row reports the point estimate in fractions per quarter, and the second row reports the corresponding standard error. Below the premia estimates, we report the R^2 statistic for a cross-sectional regression of average returns on our test assets onto the fitted values

[18] Krishnamurthy and Vissing-Jørgensen (2012) conclude, "Our finding of a convenience demand for Treasury debt suggests caution against the common practice of identifying the Treasury interest rate with models' riskless interest rate." Similar arguments can be found in Duffie and Singleton (1997) and Hull, Predescu, and White (2004).

[19] All our VAR systems forecast returns rather than cash flows. As Engsted, Pedersen, and Tanggaard (2012) clarify, results are approximately invariant to this decision, notwithstanding the concerns of Chen and Zhao (2009).

from the model as well as the J statistic. In the final two rows of each panel, we report the implied risk-aversion coefficient, γ, which can be recovered as g_1/g_2, as well as the sensitivity of news about risk to news about market variance, ω, which can be recovered as $-2g_3/g_2$.

Table 4 Panel A shows that in the 1931-1963 period, all our models explain the cross-section of stock returns reasonably well. The cross-sectional R^2 statistics are approximately 52% for both forms of our three-beta ICAPM. Both the Sharpe-Lintner and Black versions of the CAPM do a slightly poorer job describing the cross section (both R^2 statistics are roughly 50%). The two-beta ICAPM of CV (2004) performs slightly better than the CAPM and about as well as the three-beta ICAPM. Consistent with the claim that the three-beta model does a good job describing the cross-section, Table 4 shows that the constrained and the unrestricted factor model barely improve pricing relative to the three-beta ICAPM. Despite this apparent success, all models are rejected based on the standard J test. This may not be surprising, given that even the empirical three-factor model of Fama and French (1993) is rejected by this test.

Results are very different in the 1963-2011 period. Table 4 Panel B shows that in this period, both versions of the CAPM do a very poor job of explaining cross-sectional variation in average returns on portfolios sorted by size and book-to-market. When the zero-beta rate is left as a free parameter, the cross-sectional regression picks a zero-beta rate greater than the average return on the market and a negative beta premium, and implies an R^2 of 1%. When the zero-beta rate is constrained to the risk-free rate, the CAPM R^2 falls to -35%. The unconstrained zero-beta rate version of the two-beta CV (2004) model does a better job describing the cross section of average returns than the CAPM. However, the zero-beta rate is counterintuitively lower than the Treasury bill rate, and the implied coefficient of risk aversion is arguably extreme at 23.

If we restrict the zero-beta rate to equal the Treasury bill rate in our three-beta model, this model also does a poor job explaining cross-sectional variation in average returns across our test assets. However, if we allow an unrestricted zero-beta rate, the explained variation of the three-beta model increases to 63%. The estimated risk price for cash-flow beta is an economically reasonable 24.8% per year with an implied coefficient of relative risk aversion of 7.2. As before, all models are rejected based on the J statistic.

The relatively poor performance of the risk-free rate version of the three-beta ICAPM is due to the derived link between γ and ω. This is illustrated by the pricing results in Table 4 for a partially-constrained factor model that removes the constraint linking γ and ω but retains the constraints on the zero-beta rate and the discount-rate beta premium. The cross-sectional R^2 for this model increases from -37% to 71%, and the risk prices for γ and ω remain economically large and of the right sign.

The top part of Figure 4 provides a visual summary of the modern-period results. The poor performance of the CAPM in this sample period is immediately apparent. The version of the ICAPM with a restricted zero-beta rate, equal to the risk-free rate or Treasury bill rate, generates some cross-sectional spread in predicted returns that lines up qualitatively with average realized returns. However, almost all returns are underpredicted because stocks are estimated to be volatility hedges in the modern period, so the model implies a relatively low equity premium. This problem disappears when we free up the zero-beta rate in the ICAPM, adding the spread between the zero-beta rate and the Treasury bill rate to the predicted excess return over the bill rate.

6.1.1 Implications for the history of marginal utility

As a further check on the reasonableness of the model estimated in Table 4, Panel B, we can ask what the model implies for the history of our investor's marginal utility. Figure 5 plots the time-series of the exponentially smoothed combined shock $\gamma N_{CF} - N_{DR} - \frac{1}{2}\omega N_V$ based on the estimate of the three-beta model with an unrestricted zero-beta rate, The smoothed shock has correlation 0.77 with equivalently smoothed N_{CF}, 0.02 with smoothed $-N_{DR}$, and -0.80 with smoothed N_V. Figure 5 also plots the corresponding smoothed shock series for the CAPM ($N_{CF} - N_{DR}$) and for the two-beta ICAPM ($\gamma N_{CF} - N_{DR}$). The two-beta model shifts the history of good and bad times relative to the CAPM, as emphasized by CGP (2013). The model with stochastic volatility further accentuates that periods with high market volatility, such as the 1930s and the late 2000s, are particularly hard times for long-term investors.

6.2 Alternative test assets

We confirm that the success of the three-beta ICAPM is robust by expanding the set of test portfolios beyond the 25 size- and book-to-market-sorted portfolios. In particular, we add six risk-sorted portfolios and 18 characteristic- and risk-sorted assets as test assets, as well as managed versions of all of these portfolios (including the 25 characteristic-sorted assets), scaled by $EVAR$. We add five additional rows to each panel that report the cross-sectional R^2 statistics for various subsets of the test assets.

Table 5 Panel A shows that even in the 1931-1963 period, the addition of risk-sorted and managed portfolios presents a strong challenge to restricted zero-beta rate versions of all three models under consideration. Though the overall cross-sectional R^2s are relatively high, all struggle to explain some of the test asset subsets. For example, the R^2 for the original test assets (the 25 unscaled size- and book-to-market-sorted portfolios) becomes strongly negative for each of the three models. Unconstrained zero-beta rate versions of all three models perform significantly better with not only R^2s over 70% but also positive R^2s for all the test asset subsets studied.

Table 5 Panel B documents that the performance of the Black CAPM in the modern period further deteriorates when asked to price not only characteristic-sorted but also risk-sorted and managed portfolios. The unconstrained zero-beta rate version of the two-beta CV (2004) model does a better job describing the cross section of average returns than the CAPM but struggles pricing the risk-sorted assets.

The three-beta model with a restricted zero-beta rate outperforms both the Black CAPM and the unconstrained two-beta ICAPM model, delivering an overall R^2 of 47%. Of particular note, the three-beta model does a good job explaining cross-sectional variation in the average returns on the characteristic- and risk-sorted assets. These portfolios present an interesting challenge as they incorporate the idiosyncratic risk anomaly of AHXZ (2006). Of all of the three models considered, only our stochastic volatility ICAPM is able to explain the idiosyncratic anomaly effect. On the other hand, with a restricted zero-beta rate the three-beta model's explanatory power varies substantially across the test asset subsets, and in particular the model struggles when faced with either the 25 characteristic-sorted or the six risk-sorted portfolios.

If the zero-beta rate is no longer restricted, the explained variation of the three-beta model increases to 59%, with consistent pricing across the various subsets of test assets. Moreover, the freely estimated zero-beta rate is far from extreme, exceeding the Treasury Bill rate by only 40 basis points a quarter (a statistically insignificant difference).

The bottom part of Figure 4 provides a visual summary of the modern-period results with the larger set of test assets. The version of the three-beta ICAPM with a restricted zero-beta rate explains cross-sectional variation in average returns well for all of the test assets considered. This success is in stark contrast to the abysmal performance of the CAPM.

7 Conclusion

We extend the approximate closed-form intertemporal capital asset pricing model of Campbell (1993) to allow for stochastic volatility. Our model recognizes that an investor's investment opportunities may deteriorate either because expected stock returns decline or because the volatility of stock returns increases. A long-term investor with Epstein-Zin preferences and relative risk-aversion greater than one, holding an aggregate stock index, will wish to hedge against both types of changes in investment opportunities. Such an investor's perception of a stock's risk is determined not only by its beta with unexpected market returns and news about future returns (or equivalently, news about market cash flows and discount rates), but also by its beta with news about future market volatility. Although our model has three dimensions of risk, the prices of all these risks are determined by a single free parameter, the investor's coefficient of relative risk aversion.

Our implementation models the return on the aggregate stock market as one element of a vector autoregressive (VAR) system; the volatility of all shocks to the VAR is another element of the system. The empirical implementation of our VAR reveals new low-frequency movements in market volatility tied to the default spread. We show that the negative post-1963 CAPM alphas of growth stocks are justified because these stocks hedge long-term investors against both declining expected stock returns, and increasing volatility. The addition of volatility risk to the model helps it fit the cross-section of value and growth stocks, and small and large stocks, with a moderate, economically reasonable value of risk aversion.

We confront our model with portfolios of stocks sorted by past betas with the market return and volatility, and portfolios double-sorted by characteristics and past volatility betas. We also confront our model with managed portfolios that vary equity exposure in response to our estimates of market variance. The explanatory power of the model is quite good across all these sets of test assets, with stable parameter estimates. Notably, the model helps to explain the low cross-sectional reward to past market beta and the negative return to idiosyncratic volatility as the result of volatility exposures of stocks with these characteristics in the post-1963 period.

We do not interpret our model as a representative-agent model of general equilibrium in financial markets, because the model does not explain why a conservative long-term investor with constant risk aversion retains a constant equity exposure in response to changes in the equity premium that are not proportional to changes in the variance of stock returns. However, our model does answer the interesting microeconomic question: Are there reasonable preference parameters that would make a long-term investor, constrained to invest

100% in equity, content to hold the market rather than tilting towards value stocks or other high-return stock portfolios? Our answer is clearly yes.

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Table 1: VAR Estimation

The table shows the WLS parameter estimates for a first-order VAR model. The state variables in the VAR include the log real return on the CRSP value-weight index (r_M), the realized variance ($RVAR$) of within-quarter daily simple returns on the CRSP value-weight index, the log ratio of the S&P 500's price to the S&P 500's ten-year moving average of earnings (PE), the log three-month Treasury Bill yield (r_{Tbill}), the default yield spread (DEF) in percentage points, measured as the difference between the log yield on Moody's BAA bonds and the log yield on Moody's AAA bonds, and the small-stock value spread (VS), the difference in the log book-to-market ratios of small value and small growth stocks. The small-value and small-growth portfolios are two of the six elementary portfolios constructed by Davis et al. (2000). For the sake of interpretation, we estimate the VAR in two stages. Panel A reports the WLS parameter estimates of a first-stage regression forecasting $RVAR$ with the VAR state variables. The forecasted values from this regression are used in the second stage of the estimation procedure as the state variable $EVAR$, replacing $RVAR$ in the second-stage VAR. Panel B reports WLS parameter estimates of the full second-stage VAR. Initial WLS weights on each observation are inversely proportional to $RVAR_t$ and $EVAR_t$ in the first and second stages respectively and are then shrunk to equal weights so that the maximum ratio of actual weights used is less than or equal to five. Additionally, the forecasted values for both $RVAR$ and $EVAR$ are constrained to be positive. In Panels A and B, the first seven columns report coefficients on an intercept and the six explanatory variables, and the remaining column shows the implied R^2 statistic for the unscaled model. Bootstrapped standard errors that take into account the uncertainty in generating $EVAR$ are in parentheses. Panel C of the table reports the correlation ("Corr/std") matrices of both the unscaled and scaled shocks from the second-stage VAR, with shock standard deviations on the diagonal. Panel D reports the results of regressions forecasting the squared second-stage residuals from the VAR with $EVAR_t$. Bootstrap standard errors that take into account the uncertainty in generating $EVAR$ are in parentheses. The sample period for the dependent variables is 1926.3-2011.4, 342 quarterly data points.

Panel A: Forecasting Quarterly Realized Variance ($RVAR_{t+1}$)

Constant	$r_{M,t}$	$RVAR_t$	PE_t	$r_{Tbill,t}$	DEF_t	VS_t	$R^2\%$
-0.020	-0.005	0.374	0.006	-0.042	0.006	0.000	37.80%
(0.009)	(0.005)	(0.066)	(0.002)	(0.057)	(0.001)	(0.003)	

Panel B: VAR Estimates

Second stage	Constant	$r_{M,t}$	$EVAR_t$	PE_t	$r_{Tbill,t}$	DEF_t	VS_t	$R^2\%$
$r_{M,t+1}$	0.221	0.041	0.335	-0.042	-0.810	0.010	-0.051	3.36%
	(0.113)	(0.063)	(2.143)	(0.032)	(0.736)	(0.022)	(0.035)	
$EVAR_{t+1}$	-0.016	-0.002	0.441	0.005	-0.021	0.004	0.001	60.78%
	(0.007)	(0.001)	(0.057)	(0.002)	(0.046)	(0.001)	(0.002)	
PE_{t+1}	0.155	0.130	0.674	0.961	-0.399	-0.001	-0.024	94.29%
	(0.113)	(0.062)	(2.112)	(0.032)	(0.734)	(0.022)	(0.035)	
$r_{Tbill,t+1}$	0.001	0.002	-0.084	0.001	0.948	0.001	-0.001	94.07%
	(0.004)	(0.002)	(0.075)	(0.001)	(0.024)	(0.001)	(0.001)	
DEF_{t+1}	0.194	-0.293	11.162	-0.118	4.102	0.744	0.175	88.22%
	(0.309)	(0.176)	(5.838)	(0.086)	(1.925)	(0.062)	(0.094)	
VS_{t+1}	0.147	0.069	2.913	-0.017	-0.253	-0.004	0.932	93.93%
	(0.111)	(0.065)	(2.169)	(0.031)	(0.705)	(0.022)	(0.034)	

Panel C: Correlations and Standard Deviations

Corr/std	r_M	$EVAR$	PE	r_{Tbill}	DEF	VS
			unscaled			
r_M	0.105	-0.509	0.907	-0.041	-0.482	-0.039
$EVAR$	-0.509	0.004	-0.592	-0.163	0.688	0.106
PE	0.907	-0.592	0.099	-0.004	-0.598	-0.066
r_{Tbill}	-0.041	-0.163	-0.004	0.003	-0.111	0.013
DEF	-0.482	0.688	-0.598	-0.111	0.287	0.323
VS	-0.039	0.106	-0.066	0.013	0.323	0.086
			scaled			
r_M	1.138	-0.494	0.905	-0.055	-0.367	0.022
$EVAR$	-0.494	0.044	-0.570	-0.178	0.664	0.068
PE	0.905	-0.570	1.047	-0.014	-0.479	0.005
r_{Tbill}	-0.055	-0.178	-0.014	0.041	-0.160	-0.001
DEF	-0.367	0.664	-0.479	-0.160	2.695	0.273
VS	0.022	0.068	0.005	-0.001	0.273	0.996

Panel D: Heteroskedastic Shocks

Squared, second-stage, unscaled residual	Constant	$EVAR_t$	$R^2\%$
$r_{M,t+1}$	-0.002	1.85	20.43%
	(0.003)	(0.283)	
$EVAR_{t+1}$	0.000	0.004	6.36%
	(0.000)	(0.001)	
PE_{t+1}	-0.004	1.89372	19.75%
	(0.003)	(0.289)	
$r_{Tbill,t+1}$	0.000	0.000	-0.29%
	(0.000)	(0.000)	
DEF_{t+1}	-0.113	27.166	27.50%
	(0.041)	(3.411)	
VS_{t+1}	0.004	0.472	5.57%
	(0.002)	(0.133)	

Table 2: Cash-flow, Discount-rate, and Variance News for the Market Portfolio

The table shows the properties of cash-flow news (N_{CF}), discount-rate news (N_{DR}), and volatility news (N_V) implied by the VAR model of Table 1. The upper-left section of the table shows the covariance matrix of the news terms. The upper-right section shows the correlation matrix of the news terms with standard deviations on the diagonal. The lower-left section shows the correlation of shocks to individual state variables with the news terms. The lower-right section shows the functions ($\mathbf{e1}'+\mathbf{e1}'\lambda_{DR}$, $\mathbf{e1}'\lambda_{DR}$, $\mathbf{e2}'\lambda_V$) that map the state-variable shocks to cash-flow, discount-rate, and variance news. We define $\lambda_{DR} \equiv \rho\mathbf{\Gamma}(\mathbf{I}-\rho\mathbf{\Gamma})^{-1}$ and $\lambda_V \equiv \rho(\mathbf{I}-\rho\mathbf{\Gamma})^{-1}$, where $\mathbf{\Gamma}$ is the estimated VAR transition matrix from Table 1 and ρ is set to 0.95 per annum. r_M is the log real return on the CRSP value-weight index. $RVAR$ is the realized variance of within-quarter daily simple returns on the CRSP value-weight index. PE is the log ratio of the S&P 500's price to the S&P 500's ten-year moving average of earnings. r_{Tbill} is the log three-month Treasury Bill yield. DEF is the default yield spread in percentage points, measured as the difference between the log yield on Moody's BAA bonds and the log yield on Moody's AAA bonds. VS is the small-stock value-spread, the difference in the log book-to-market ratios of small value and small growth stocks. Bootstrap standard errors that take into account the uncertainty in generating $EVAR$ are in parentheses.

News cov.	N_{CF}	N_{DR}	N_V	News corr/std	N_{CF}	N_{DR}	N_V
N_{CF}	0.00236	-0.00018	-0.00015	N_{CF}	0.049	-0.041	-0.121
	(0.00087)	(0.00119)	(0.00030)		(0.008)	(0.225)	(0.264)
N_{DR}	-0.00018	0.00838	-0.00008	N_{DR}	-0.041	0.092	-0.034
	(0.00119)	(0.00270)	(0.00065)		(0.225)	(0.014)	(0.355)
N_V	-0.00015	-0.00008	0.00065	N_V	-0.121	-0.034	0.025
	(0.00030)	(0.00065)	(0.00030)		(0.264)	(0.355)	(0.007)

Shock corr.	N_{CF}	N_{DR}	N_V	Functions	N_{CF}	N_{DR}	N_V
r_M shock	0.497	-0.888	-0.026	r_M shock	0.908	-0.092	-0.011
	(0.213)	(0.045)	(0.332)		(0.031)	(0.031)	(0.015)
$EVAR$ shock	-0.001	0.472	0.452	$RVAR$ shock	-0.300	-0.300	1.280
	(0.168)	(0.113)	(0.180)		(1.134)	(1.134)	(0.571)
PE shock	0.158	-0.960	-0.097	PE shock	-0.814	-0.814	0.187
	(0.239)	(0.044)	(0.354)		(0.167)	(0.167)	(0.084)
r_{Tbill} shock	-0.372	-0.151	-0.034	r_{Tbill} shock	-4.245	-4.245	0.867
	(0.219)	(0.142)	(0.331)		(3.635)	(3.635)	(1.821)
DEF shock	-0.041	0.533	0.751	DEF shock	0.008	0.008	0.079
	(0.188)	(0.115)	(0.223)		(0.034)	(0.034)	(0.017)
VS shock	-0.397	-0.165	0.567	VS shock	-0.248	-0.248	0.099
	(0.187)	(0.141)	(0.261)		(0.127)	(0.127)	(0.064)

Table 3: Cash-flow, Discount-rate, and Variance Betas

The table shows the estimated cash-flow ($\widehat{\beta}_{CF}$), discount-rate ($\widehat{\beta}_{DR}$), and variance betas ($\widehat{\beta}_{V}$) for the 25 ME- and BE/ME-sorted portfolios (Panels A and B) and six risk-sorted portfolios (Panels C and D) for the early (1931:3-1963:2) and modern (1963:3-2011:4) subsamples respectively. “Growth” denotes the lowest BE/ME, “Value” the highest BE/ME, “Small” the lowest ME, and "Large" the highest ME stocks. $\widehat{b}_{\Delta VAR}$ and $\widehat{b}_{r_M}$ are past return-loadings on the weighted sum of changes in the VAR state variables, where the weights are according to λ_V as estimated in Table 2, and on the market-return shock. “Diff.” is the difference between the extreme cells. Bootstrapped standard errors [in brackets] are conditional on the estimated news series. Estimates are based on quarterly data using weighted least squares where the weights are the same as those used to estimate the VAR.

25 ME- and BE/ME-sorted portfolios

Panel A: Early Period (1931:3-1963:2)

$\widehat{\beta}_{CF}$	Growth		2		3		4		Value		Diff	
Small	0.49	[0.13]	0.42	[0.11]	0.44	[0.11]	0.44	[0.10]	0.46	[0.10]	-0.04	[0.05]
2	0.30	[0.08]	0.36	[0.09]	0.37	[0.09]	0.39	[0.09]	0.42	[0.10]	0.12	[0.04]
3	0.32	[0.08]	0.29	[0.08]	0.34	[0.09]	0.33	[0.08]	0.47	[0.12]	0.15	[0.05]
4	0.26	[0.07]	0.28	[0.08]	0.31	[0.09]	0.35	[0.08]	0.44	[0.11]	0.18	[0.05]
Large	0.24	[0.07]	0.23	[0.07]	0.27	[0.09]	0.34	[0.10]	0.40	[0.29]	0.16	[0.04]
Diff	-0.26	[0.07]	-0.19	[0.05]	-0.17	[0.04]	-0.10	[0.03]	-0.06	[0.03]		

$\widehat{\beta}_{DR}$	Growth		2		3		4		Value		Diff	
Small	1.20	[0.15]	1.21	[0.16]	1.20	[0.17]	1.19	[0.17]	1.13	[0.17]	-0.07	[0.07]
2	0.87	[0.11]	1.03	[0.14]	1.01	[0.15]	0.99	[0.16]	1.14	[0.14]	0.27	[0.08]
3	0.95	[0.13]	0.81	[0.09]	0.97	[0.12]	0.93	[0.12]	1.22	[0.16]	0.27	[0.09]
4	0.67	[0.07]	0.81	[0.10]	0.85	[0.10]	0.93	[0.14]	1.24	[0.17]	0.58	[0.13]
Large	0.70	[0.08]	0.66	[0.08]	0.80	[0.12]	1.05	[0.16]	0.90	[0.12]	0.20	[0.13]
Diff	-0.50	[0.14]	-0.56	[0.11]	-0.40	[0.16]	-0.13	[0.13]	-0.23	[0.08]		

$\widehat{\beta}_{V}$	Growth		2		3		4		Value		Diff	
Small	-0.14	[0.05]	-0.14	[0.04]	-0.15	[0.05]	-0.14	[0.04]	-0.14	[0.04]	0.00	[0.02]
2	-0.08	[0.03]	-0.10	[0.03]	-0.10	[0.03]	-0.11	[0.03]	-0.14	[0.04]	-0.06	[0.02]
3	-0.09	[0.03]	-0.07	[0.02]	-0.09	[0.03]	-0.10	[0.03]	-0.14	[0.04]	-0.05	[0.02]
4	-0.04	[0.02]	-0.06	[0.02]	-0.08	[0.03]	-0.10	[0.04]	-0.15	[0.05]	-0.10	[0.03]
Large	-0.05	[0.02]	-0.05	[0.02]	-0.09	[0.04]	-0.12	[0.04]	-0.11	[0.03]	-0.07	[0.03]
Diff	0.09	[0.04]	0.09	[0.02]	0.06	[0.02]	0.02	[0.02]	0.03	[0.02]		

Panel B: Modern Period (1963:3-2011:4)

$\widehat{\beta}_{CF}$	Growth		2		3		4		Value		Diff	
Small	0.23	[0.06]	0.24	[0.05]	0.26	[0.05]	0.25	[0.04]	0.28	[0.05]	0.05	[0.04]
2	0.23	[0.06]	0.24	[0.05]	0.26	[0.05]	0.27	[0.05]	0.29	[0.05]	0.05	[0.04]
3	0.21	[0.05]	0.25	[0.05]	0.24	[0.05]	0.25	[0.05]	0.27	[0.05]	0.06	[0.03]
4	0.21	[0.05]	0.24	[0.04]	0.25	[0.04]	0.25	[0.04]	0.28	[0.05]	0.07	[0.03]
Large	0.15	[0.04]	0.20	[0.03]	0.18	[0.03]	0.20	[0.04]	0.20	[0.04]	0.05	[0.03]
Diff	-0.08	[0.04]	-0.04	[0.03]	-0.08	[0.03]	-0.05	[0.03]	-0.07	[0.03]		

$\widehat{\beta}_{DR}$	Growth		2		3		4		Value		Diff	
Small	1.30	[0.11]	1.05	[0.09]	0.87	[0.07]	0.81	[0.07]	0.86	[0.09]	-0.44	[0.08]
2	1.19	[0.09]	0.94	[0.08]	0.82	[0.07]	0.74	[0.07]	0.80	[0.08]	-0.39	[0.08]
3	1.11	[0.08]	0.87	[0.06]	0.73	[0.06]	0.70	[0.07]	0.69	[0.07]	-0.42	[0.08]
4	1.00	[0.07]	0.82	[0.06]	0.73	[0.07]	0.70	[0.07]	0.75	[0.07]	-0.26	[0.08]
Large	0.82	[0.05]	0.68	[0.04]	0.60	[0.05]	0.59	[0.07]	0.64	[0.06]	-0.18	[0.06]
Diff	-0.48	[0.10]	-0.37	[0.08]	-0.26	[0.06]	-0.22	[0.07]	-0.23	[0.08]		

$\widehat{\beta}_{V}$	Growth		2		3		4		Value		Diff	
Small	0.13	[0.07]	0.08	[0.06]	0.05	[0.05]	0.05	[0.05]	0.01	[0.07]	-0.13	[0.03]
2	0.14	[0.06]	0.08	[0.06]	0.05	[0.05]	0.04	[0.05]	0.03	[0.06]	-0.12	[0.02]
3	0.14	[0.06]	0.07	[0.05]	0.05	[0.05]	0.02	[0.05]	0.04	[0.04]	-0.10	[0.03]
4	0.13	[0.05]	0.07	[0.05]	0.03	[0.05]	0.02	[0.06]	0.01	[0.06]	-0.11	[0.02]
Large	0.09	[0.05]	0.07	[0.04]	0.03	[0.04]	0.02	[0.05]	0.02	[0.04]	-0.08	[0.02]
Diff	-0.04	[0.03]	-0.01	[0.03]	-0.02	[0.02]	-0.03	[0.02]	0.01	[0.03]		

6 risk-sorted portfolios

Panel C: Early Period (1931:3-1963:2)

$\widehat{\beta}_{CF}$	Lo $\widehat{b}_{r_M}$		2		Hi $\widehat{b}_{r_M}$		Diff	
Lo $\widehat{b}_{VAR}$	0.23	[0.07]	0.34	[0.09]	0.42	[0.11]	0.19	[0.04]
Hi $\widehat{b}_{VAR}$	0.21	[0.06]	0.28	[0.08]	0.41	[0.11]	0.20	[0.05]
Diff	-0.02	[0.02]	-0.05	[0.03]	-0.01	[0.02]		

$\widehat{\beta}_{DR}$	Lo $\widehat{b}_{r_M}$		2		Hi $\widehat{b}_{r_M}$		Diff	
Lo $\widehat{b}_{VAR}$	0.60	[0.06]	0.89	[0.11]	1.13	[0.13]	0.54	[0.11]
Hi $\widehat{b}_{VAR}$	0.58	[0.07]	0.83	[0.10]	1.11	[0.16]	0.54	[0.13]
Diff	-0.02	[0.04]	-0.06	[0.08]	-0.02	[0.06]		

$\widehat{\beta}_{V}$	Lo $\widehat{b}_{r_M}$		2		Hi $\widehat{b}_{r_M}$		Diff	
Lo $\widehat{b}_{VAR}$	-0.04	[0.02]	-0.07	[0.03]	-0.10	[0.04]	-0.06	[0.02]
Hi $\widehat{b}_{VAR}$	-0.05	[0.02]	-0.07	[0.03]	-0.11	[0.04]	-0.06	[0.03]
Diff	-0.01	[0.02]	0.00	[0.02]	-0.01	[0.02]		

Panel D: Modern Period (1963:3-2011:4)

$\widehat{\beta}_{CF}$	Lo $\widehat{b}_{r_M}$		2		Hi $\widehat{b}_{r_M}$		Diff	
Lo $\widehat{b}_{VAR}$	0.20	[0.04]	0.20	[0.04]	0.26	[0.06]	0.06	[0.04]
Hi $\widehat{b}_{VAR}$	0.17	[0.03]	0.21	[0.04]	0.21	[0.06]	0.05	[0.05]
Diff	-0.04	[0.03]	0.01	[0.02]	-0.05	[0.02]		

$\widehat{\beta}_{DR}$	Lo $\widehat{b}_{r_M}$		2		Hi $\widehat{b}_{r_M}$		Diff	
Lo $\widehat{b}_{VAR}$	0.63	[0.06]	0.79	[0.06]	1.18	[0.09]	0.56	[0.08]
Hi $\widehat{b}_{VAR}$	0.58	[0.06]	0.85	[0.05]	1.24	[0.09]	0.66	[0.11]
Diff	-0.04	[0.09]	0.06	[0.06]	0.06	[0.05]		

$\widehat{\beta}_{V}$	Lo $\widehat{b}_{r_M}$		2		Hi $\widehat{b}_{r_M}$		Diff	
Lo $\widehat{b}_{VAR}$	0.04	[0.05]	0.06	[0.05]	0.09	[0.07]	0.05	[0.03]
Hi $\widehat{b}_{VAR}$	0.06	[0.04]	0.09	[0.05]	0.12	[0.07]	0.06	[0.04]
Diff	0.02	[0.02]	0.03	[0.02]	0.03	[0.02]		

Table 4: Asset Pricing Tests: 25 Size and Book-to-Market Portfolios

The table reports GMM estimates of the CAPM, the 2-beta ICAPM, the 3-beta volatility ICAPM, a factor model where only the $\widehat{\beta}_{DR}$ premium is restricted, and an unrestricted factor model for the early (Panel A: 1931:3-1963:2) and modern (Panel B: 1963:3-2011:4) subsamples. The test assets are 25 ME- and BE/ME-sorted portfolios. The first column per model constrains the zero-beta rate (R_{zb}) to equal the T-bill rate (R_{Tbill}) while the second column allows R_{zb} to be a free parameter. The 5% critical value for the test of overidentifying restrictions is 36.5 in columns 1, 3, and 5; 35.2 in columns 2, 4, 6, and 7; 34.0 in columns 8 and 9; and 32.7 in column 10.

Parameter	CAPM		2-beta ICAPM		3-beta ICAPM		Constrained		Unrestricted	
Panel A: Early Period										
R_{zb} less R_f (g_0)	0	0.001	0	0.002	0	0.003	0	0.007	0	0.020
Std. err.	0	(0.015)	0	(0.013)	0	(0.012)	0	(0.014)	0	(0.018)
$\widehat{\beta}_{CF}$ premium (g_1)	0.035	0.034	0.088	0.081	0.075	0.070	0.069	0.031	0.081	0.066
Std. err.	(0.012)	(0.018)	(0.048)	(0.065)	(0.033)	(0.042)	(0.053)	(0.072)	(0.066)	(0.071)
$\widehat{\beta}_{DR}$ premium (g_2)	0.035	0.034	0.016	0.016	0.016	0.016	0.016	0.016	0.010	-0.025
Std. err.	(0.012)	(0.018)	0	0	0	0	(0.000)	(0.000)	(0.027)	(0.035)
$\widehat{\beta}_{VAR}$ premium (g_3)					-0.040	-0.032	-0.063	-0.124	-0.067	-0.259
Std. err.					(0.050)	(0.055)	(0.143)	(0.144)	(0.155)	(0.192)
$\widehat{R}^2$	50%	50%	52%	52%	52%	52%	52%	53%	52%	56%
J statistic	49.1	46.6	54.4	51.5	51.4	49.6	49.4	40.5	50.3	34.6
Implied γ	2.2	2.2	5.5	5.1	4.8	4.4	N/A	N/A	N/A	N/A
Implied ω	N/A	N/A	N/A	N/A	5.0	4.0	N/A	N/A	N/A	N/A
Panel B: Modern Period										
R_{zb} less R_f (g_0)	0	0.026	0	-0.027	0	0.008	0	-0.012	0	-0.012
Std. err.	0	(0.010)	0	(0.013)	0	(0.009)	0	(0.014)	0	(0.015)
$\widehat{\beta}_{CF}$ premium (g_1)	0.020	-0.003	0.068	0.178	0.052	0.055	0.091	0.136	0.121	0.160
Std. err.	(0.008)	(0.012)	(0.035)	(0.061)	(0.014)	(0.000)	(0.037)	(0.069)	(0.036)	(0.052)
$\widehat{\beta}_{DR}$ premium (g_2)	0.020	-0.003	0.008	0.008	0.008	0.008	0.008	0.008	-0.005	-0.004
Std. err.	(0.008)	(0.012)	0	0	0	0	(0.000)	(0.000)	(0.017)	(0.020)
$\widehat{\beta}_{VAR}$ premium (g_3)					-0.062	-0.096	-0.103	-0.083	-0.047	-0.035
Std. err.					(0.079)	(0.043)	(0.048)	(0.064)	(0.067)	(0.068)
$\widehat{R}^2$	-35%	1%	29%	56%	-37%	63%	71%	75%	73%	76%
J statistic	93.4	79.5	68.2	48.4	70.6	54.2	53.9	46.8	55.6	46.3
Implied γ	2.6	-0.4	8.7	23.0	6.8	7.2	N/A	N/A	N/A	N/A
Implied ω	N/A	N/A	N/A	N/A	16.0	24.9	N/A	N/A	N/A	N/A

Table 5: Asset Pricing Tests: Adding Risk-sorted and Managed Portfolios

The table reports GMM estimates of the CAPM, the 2-beta ICAPM, the 3-beta volatility ICAPM, a factor model where only the $\widehat{\beta}_{DR}$ premium is restricted, and an unrestricted factor model for the early (Panel A: 1931:3-1963:2) and modern (Panel B: 1963:3-2011:4) subsamples. The test assets are 25 ME- and BE/ME-sorted portfolios ("char."), six risk-sorted portfolios ("risk"), and 18 characteristic and risk-sorted assets ("char./risk"). We include both "unscaled" and "managed" versions of these portfolios in our pricing tests. For the managed portfolios, we scale the test assets by $EVAR$. Thus, each column of estimates results from pricing the returns on 98 portfolios. The first column per model constrains the zero-beta rate (R_{zb}) to equal the risk-free rate (R_{Tbill}) while the second column allows R_{zb} to be a free parameter. The 5% critical value for the test of overidentifying restrictions is 121.0 in columns 1, 3, and 5; 119.9 in columns 2, 4, 6, and 7; 118.8 in columns 8 and 9; and 117.7 in column 10. The ninth row reports the cross-sectional R^2, while rows 13 through 17 report the R^2 for various test asset subsets.

Parameter	CAPM		2-beta ICAPM		3-beta ICAPM		Constrained		Unrestricted	
					Panel A: Early Period					
R_{zb} less R_f (g_0)	0	0.014	0	0.010	0	0.011	0	0.012	0	0.019
Std. err.	0	(0.015)	0	(0.017)	0	(0.016)	0	(0.013)	0	(0.016)
$\widehat{\beta}_{CF}$ premium (g_1)	0.024	0.020	0.051	0.038	0.046	0.036	0.078	0.029	0.166	0.105
Std. err.	(0.013)	(0.017)	(0.058)	(0.076)	(0.045)	(0.063)	(0.050)	(0.053)	(0.071)	(0.078)
$\widehat{\beta}_{DR}$ premium (g_2)	0.024	0.020	0.016	0.016	0.016	0.016	0.016	0.016	-0.032	-0.039
Std. err.	(0.013)	(0.017)	0	0	0	0	(0.000)	(0.000)	(0.024)	(0.021)
$\widehat{\beta}_{VAR}$ premium (g_3)					-0.009	-0.004	0.063	-0.016	-0.085	-0.229
Std. err.					(0.029)	(0.025)	(0.147)	(0.112)	(0.173)	(0.145)
$\widehat{R^2}$	71%	75%	75%	77%	74%	77%	76%	77%	82%	84%
J statistic	673.0	617.1	728.1	641.8	721.6	641.1	755.9	632.0	772.0	646.9
Implied γ	1.5	1.3	3.2	2.4	2.9	2.3	N/A	N/A	N/A	N/A
Implied ω	N/A	N/A	N/A	N/A	1.2	0.5	N/A	N/A	N/A	N/A
$\widehat{R^2}$: 25 unscaled char.	-112%	11%	-72%	6%	-82%	6%	-25%	6%	0%	45%
$\widehat{R^2}$: 49 unscaled	-71%	33%	-35%	31%	-44%	31%	7%	30%	27%	64%
$\widehat{R^2}$: 49 managed	63%	59%	66%	62%	66%	62%	62%	62%	73%	74%
$\widehat{R^2}$: 6 unscaled risk	-53%	82%	-10%	79%	-25%	79%	53%	79%	28%	72%
$\widehat{R^2}$: 18 unscaled char./risk	-49%	44%	-14%	45%	-22%	45%	26%	44%	51%	83%

Parameter	CAPM		2-beta ICAPM		3-beta ICAPM		Constrained		Unrestricted	
Panel B: Modern Period										
R_{zb} less R_f (g_0)	0	0.015	0	-0.013	0	0.004	0	-0.007	0	0.002
Std. err.	0	(0.010)	0	(0.014)	0	(0.014)	0	(0.016)	0	(0.009)
$\widehat{\beta}_{CF}$ premium (g_1)	0.017	0.006	0.063	0.118	0.055	0.055	0.073	0.100	0.104	0.098
Std. err.	(0.008)	(0.013)	(0.043)	(0.072)	(0.002)	(0.000)	(0.059)	(0.038)	(0.026)	(0.034)
$\widehat{\beta}_{DR}$ premium (g_2)	0.017	0.006	0.008	0.008	0.008	0.008	0.008	0.008	0.000	0.000
Std. err.	(0.008)	(0.013)	0	0	0	0	(0.000)	(0.000)	(0.014)	(0.015)
$\widehat{\beta}_{VAR}$ premium (g_3)					-0.086	-0.096	-0.097	-0.094	-0.096	-0.097
Std. err.					(0.036)	(0.059)	(0.053)	(0.058)	(0.048)	(0.047)
$\widehat{R^2}$	-10%	1%	19%	25%	47%	59%	62%	63%	68%	68%
J statistic	447.5	408.9	350.5	296.6	450.7	407.0	386.4	331.8	295.6	294.4
Implied γ	2.2	0.7	8.1	15.3	7.1	7.2	N/A	N/A	N/A	N/A
Implied ω	N/A	N/A	N/A	N/A	22.5	24.9	N/A	N/A	N/A	N/A
$\widehat{R^2}$: 25 unscaled char.	-58%	-7%	26%	47%	-52%	28%	39%	44%	56%	56%
$\widehat{R^2}$: 49 unscaled	-31%	-8%	17%	35%	16%	53%	58%	61%	71%	71%
$\widehat{R^2}$: 49 managed	-2%	2%	16%	16%	63%	61%	62%	63%	65%	65%
$\widehat{R^2}$: 6 unscaled risk	-24%	-50%	-9%	17%	-31%	49%	52%	44%	51%	50%
$\widehat{R^2}$: 18 unscaled char./risk	-32%	-17%	4%	22%	48%	62%	65%	67%	78%	78%

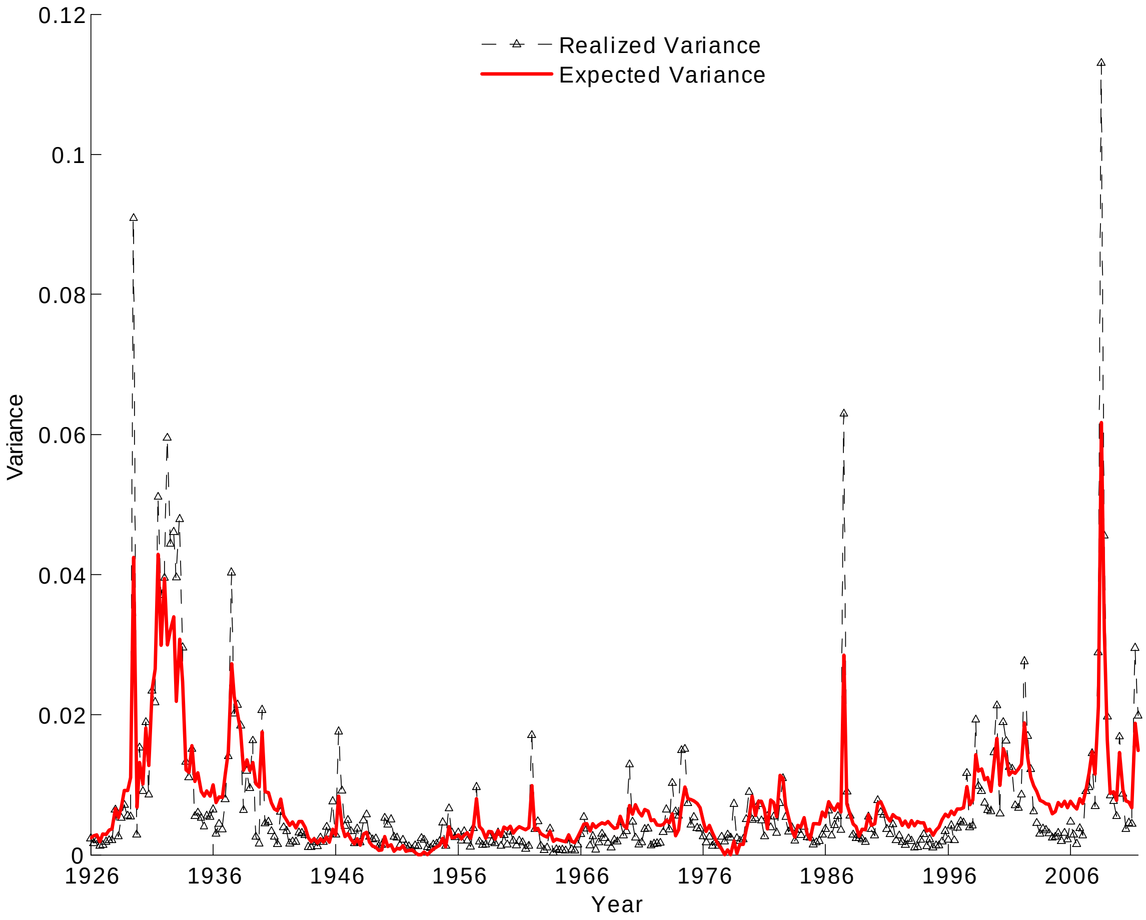

Figure 1: This figure plots quarterly observations of realized within-quarter daily return variance over the sample period 1926:2-2011:4 and the expected variance implied by the model estimated in Table 1 Panel A.

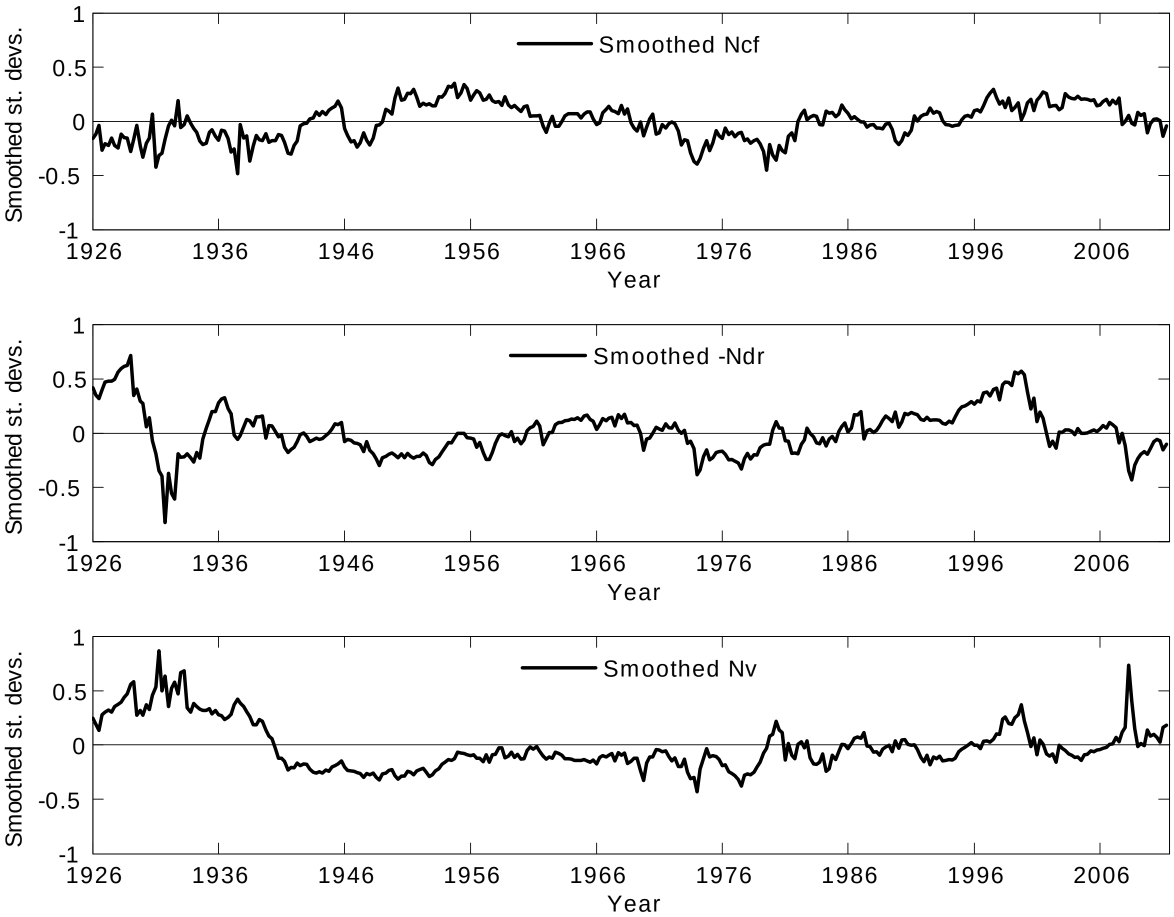

Figure 2: This figure plots normalized cash-flow news, the negative of normalized discount-rate news, and normalized variance news. The series are smoothed with a trailing exponentially-weighted moving average where the decay parameter is set to 0.08 per quarter, and the smoothed news series is generated as $MA_t(N) = 0.08N_t + (1-0.08)MA_{t-1}(N)$. This decay parameter implies a half-life of six years. The sample period is 1926:2-2011:4.

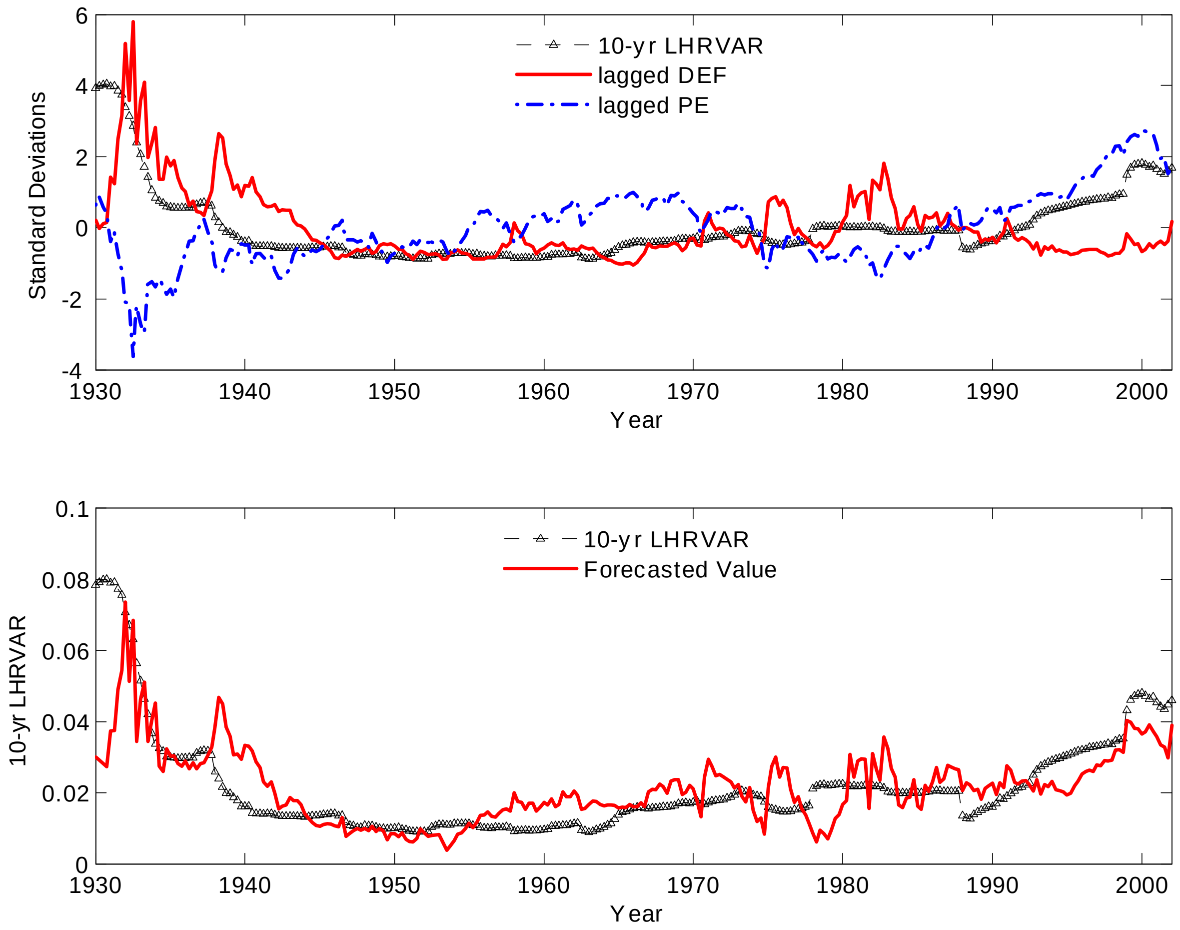

Figure 3: We measure long-horizon realized variance ($LHRVAR$) as the annualized discounted sum of within-quarter daily return variance, $LHRVAR_h = \frac{4*\Sigma_{j=1}^{h}\rho^{j-1}RVAR_{t+j}}{\Sigma_{j=1}^{h}\rho^{j-1}}$. Each panel of this figure plots quarterly observations of ten-year realized variance, $LHRVAR_{40}$, over the sample period 1930:1-2001:1. In Panel A, in addition to $LHRVAR_{40}$, we also plot lagged PE and DEF. In Panel B, in addition to $LHRVAR_{40}$, we also plot the fitted value from a regression forecasting $LHRVAR_{40}$ with $DEFO$, defined as DEF orthogonalized to demeaned PE. The appendix reports the WLS estimates of this forecasting regression.

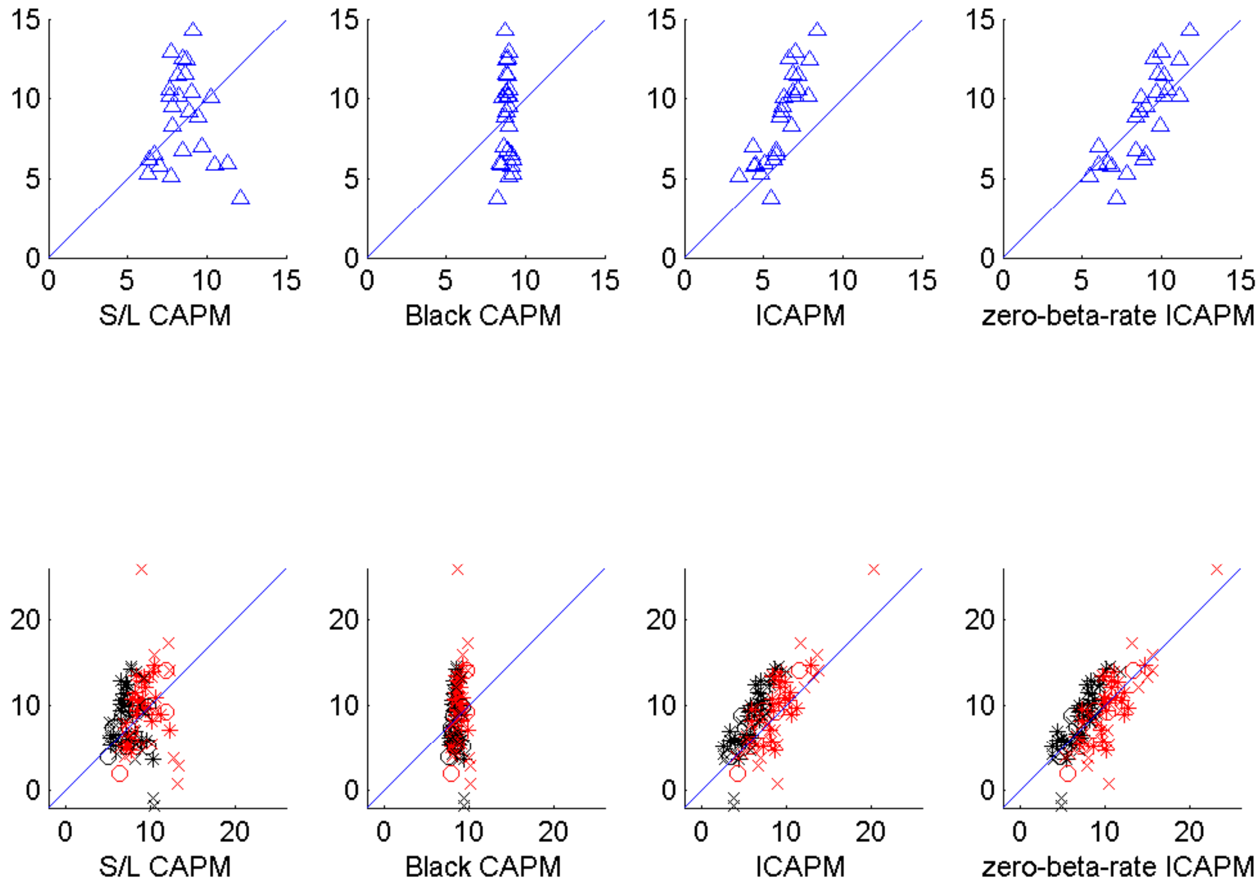

Figure 4: Each diagram plots sample against predicted average excess returns. Test assets in the top row are the 25 ME- and BE/ME-sorted portfolios and in the bottom row, both unscaled (black) and scaled by $EVAR$ (red) versions of the 25 ME- and BE/ME-sorted portfolios (asterisks), six risk-sorted portfolios (circles), and 18 characteristic- and risk-sorted portfolios (crosses). Predicted values are from Table 4 (top row) and Table 5 (bottom row) for 1963:3-2011:4. From left to right, the models tested are the Sharpe/Lintner CAPM, the Black CAPM, the three-factor ICAPM with the zero-beta rate constrained to the risk-freee rate, and the three-factor ICAPM with a free zero-beta rate.

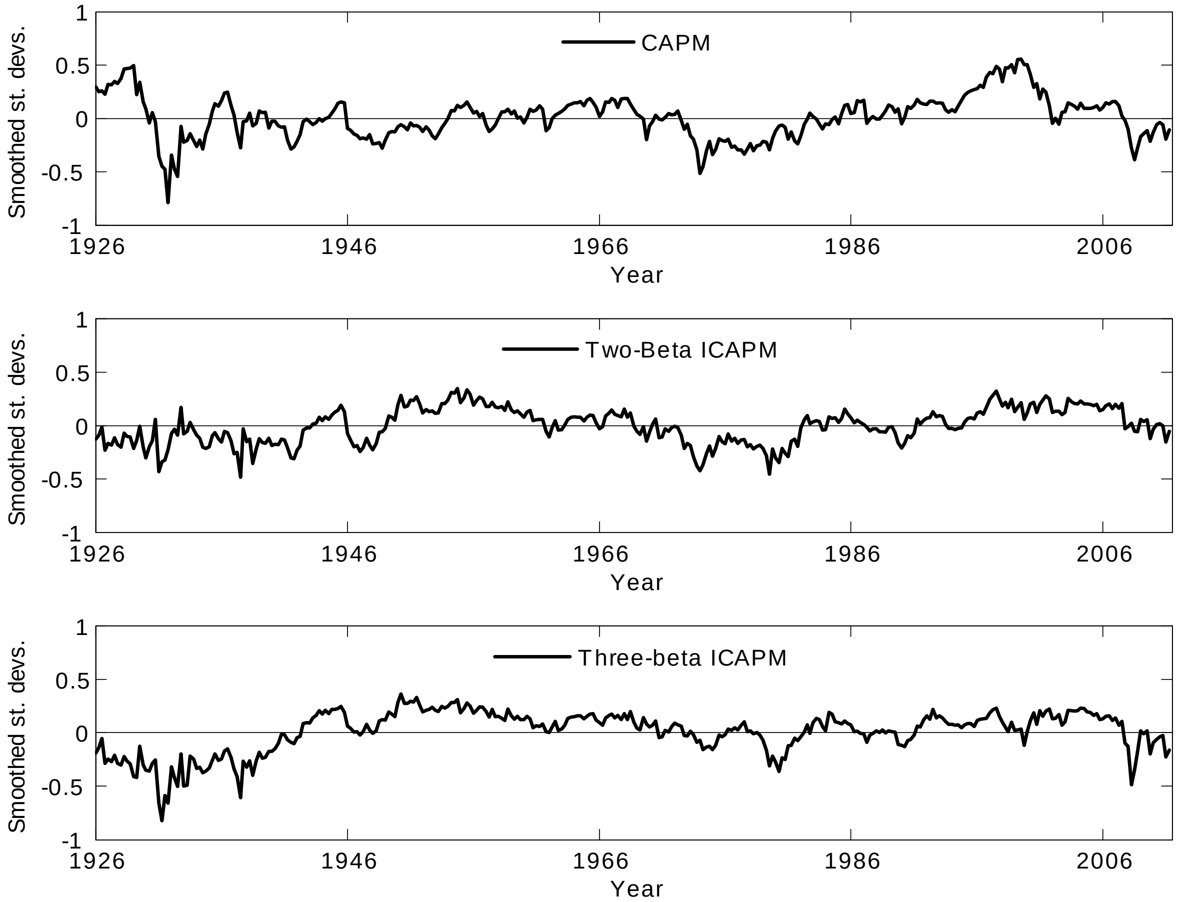

Figure 5: This figure plots the time-series of the smoothed combined shock for the CAPM ($N_{CF} - N_{DR}$), the two-beta ICAPM ($\gamma N_{CF} - N_{DR}$), and the three-beta ICAPM that includes stochastic volatility ($\gamma N_{CF} - N_{DR} - \frac{1}{2}\omega N_V$) for the unconstrained zero-beta rate specifications estimated in Table 4 Panel B for the sample period 1963:3-2011:4. The shock is smoothed with a trailing exponentially-weighted moving average. The decay parameter is set to 0.08 per quarter, and the smoothed news series is generated as $MA_t(SDF) = 0.08SDF_t + (1 - 0.08)MA_{t-1}(N)$. This decay parameter implies a half-life of approximately two years.