

**A Comparison of Parametric Models of Income Distribution**

**Across Countries and Over Time**

**Ripsy Bandourian**

**James B. McDonald**

**Robert S. Turley**

**Department of Economics**

**Brigham Young University**

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## **Abstract**

The five-parameter generalized beta distribution and ten of its special cases are considered as models for the size distribution of income. The models are fit to income data for 23 countries and various years—a total of 82 data sets. Of the models considered, the Weibull, Dagum and generalized beta of the second kind are best fitting of the models with two, three and four parameters for 62 percent, 84 percent, and 96 percent of the data sets, respectively. Increasing inequality with respect to pre-tax income is observed in most of the countries considered.

## I. Introduction

Vilfredo Pareto first proposed a model of income distribution in the form of a probability density function (pdf) in 1895. Pareto's analysis of income inequality, based on his model and economic data he gathered, stirred up the debate on the effect of economic growth on income inequality. In today's terms, are the rich getting richer and the poor getting poorer, or is the rising economic tide lifting all the boats? Corrado Gini disagreed with Pareto's opinion that economic growth leads to less inequality. To better study the phenomena, he proposed a unit-free measure of income inequality known as the Gini coefficient that is still commonly used today (Gini, 1912).

Later empirical studies showed that Pareto's distribution accurately modeled high levels of income, but did a poor job describing the low end of the distribution. As research continued, new distributions were proposed that better fit the observed data. Gibrat (1931) wrote a famous paper in which he suggested the two-parameter lognormal distribution, further examined by Aitchinson and Brown (1969). Ammon (1895) proposed the gamma distribution, which was more recently reintroduced and fit to US income data by Salem and Mount (1974). Bartels and van Metelel (1975) suggested another two-parameter pdf, the Weibull.

Even better fits could be obtained using three-parameter distributions. These include the generalized gamma (Amoroso, 1924-25 and Taille, 1981) and beta (Thurow, 1970) as well as two closely related models which are members of the Burr family of distributions: the Singh-Maddala (1976), known in statistics literature as the Burr 12, and the Dagum (1977), known as the Burr 3. These distributions allowed for intersecting Lorenz curves, a phenomenon observed in the data that could not be modeled by any of the two-parameter distributions considered.

McDonald (1984) introduced the generalized beta of the first and second kinds (GB1 and GB2), two four-parameter distributions that were not only very successful in fitting the data, but also included all of the previously mentioned distributions as special or limiting cases.

McDonald and Mantrala (1996) found that the GB2 distribution provided a significantly better fit than its nested distributions when fit to income data from the United States. The empirical success of the GB2 was complemented by Parker's (1999) theoretical model of income generation, showing earnings to follow a GB2 distribution.

This paper compares many of the probability density functions that have been considered successful in describing the size distribution of income. While others have shown the goodness-of-fit of their distributional models using US income data, one may imagine that income in other countries is distributed very differently, particularly in countries formerly Communist or in earlier stages of economic development. By comparing a diverse set of countries, it is easier to draw conclusions about the quality of these models in describing the distribution of income generally and not just the characteristics observed in the United States. Section II of this paper discusses the methodology and functional forms used in this study, the data sets being considered are described in section III, the results are presented in section IV, and section V contains the authors' conclusions.

## II. Methodology

### A. The Generalized Beta Distribution Family

McDonald and Xu (1995) showed that all of the distributions previously mentioned could be represented as special or limiting cases of a very flexible, five-parameter distribution they termed the generalized beta (GB), whose probability distribution function can be written:

$$GB(y; a, b, c, p, q) = \frac{|a|y^{ap-1}(1 - (1 - c)(y/b)^a)^{q-1}}{b^{ap}B(p, q)(1 + c(y/b)^a)^{p+q}} \text{ for } 0 < y^a < b^a$$

and zero otherwise, where  $0 \leq c \leq 1$ , and  $b, p, q > 0$ . Using this parameterization, it is easy to see the relationships within the generalized beta family of distributions. The GB2 can be derived by setting the  $c$  parameter equal to one.

$$GB2(y; a, b, p, q) = \frac{|a|y^{ap-1}(1 - (y/b)^a)^{q-1}}{b^{ap}B(p, q)} = GB(y; a, b, c=1, p, q)$$

The three-parameter Dagum and Singh-Maddala distributions correspond to the case

$$DAGUM(y; a, b, p) = GB2(y; a, b, p, q=1),$$

$$SM(y; a, b, q) = GB2(y; a, b, p=1, q).$$

The generalized gamma (GG) distribution is a limiting case defined as

$$GG(y; a, \beta, p) = \lim_{q \rightarrow \infty} GB(y; a, b = q^{1/a} \beta, c = 1, p, q).$$

A convenient way to visualize these relationships is the distribution tree in figure 1.

These relationships can be seen in greater detail in McDonald and Xu (1995).

## Models for the Distribution of Income

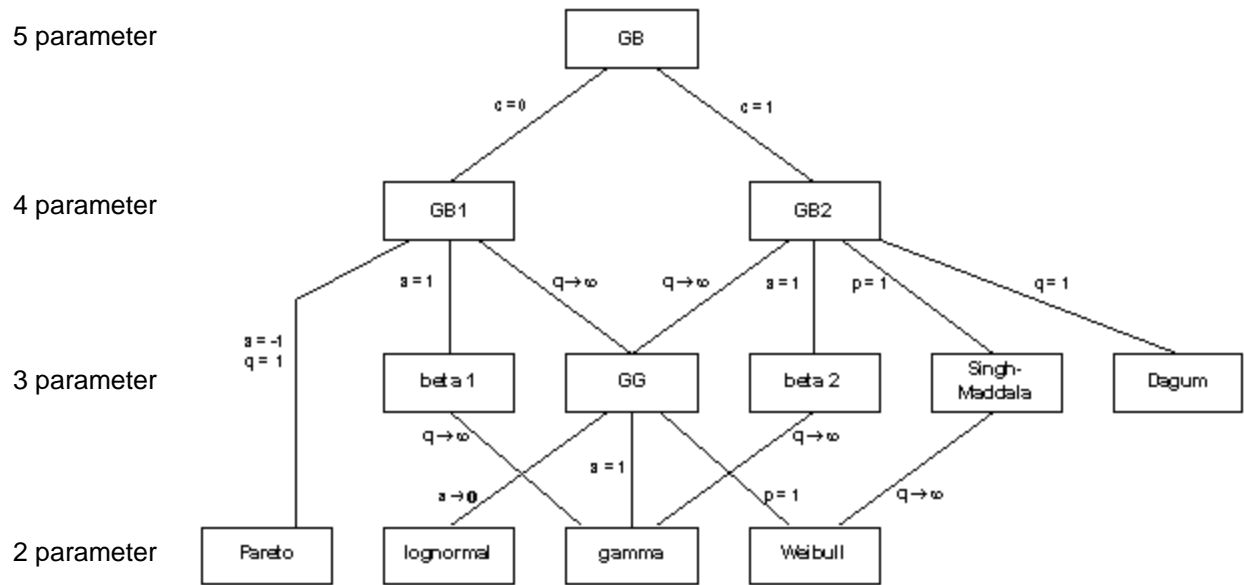


Figure 1

### B. Distributional Estimation

Since the income data utilized in this study are grouped into twenty equal probability intervals, the distributional parameters are estimated using maximum likelihood techniques by maximizing the multinomial likelihood function

$$L(\theta) = N! \prod_{i=1}^{20} \frac{(p_i(\theta))^{n_i}}{n_i!}$$

where  $p_i(\theta) = F(Y_i; \theta) - F(Y_{i-1}; \theta)$ ,  $F(\cdot)$  denotes the cumulative distribution function,  $\theta$  is a vector containing the distributional parameters,  $Y_i$  and  $Y_{i-1}$  are the upper and lower bounds of the  $i^{\text{th}}$  of twenty data groups,  $n_i$  is the number of observations in the  $i^{\text{th}}$  group, and  $N$  is the total number of observations.

There are no closed form solutions for the value of  $\theta$  that maximizes the likelihood function so iterative techniques must be used. In this paper, the likelihood function is maximized using a simplex method from the Matlab optimization toolkit, finding the minimum to the negative log likelihood function with a convergence criterion of  $10^{-8}$ .

### C. Goodness-of-Fit

In order to evaluate relative performance of the eleven distributions, we evaluate the sum of squared errors (SSE), sum of absolute errors (SAE), and chi-square( $\chi^2$ ) goodness-of-fit measures in addition to the log-likelihood value for the estimated model and data set. The SSE, SAE and  $\chi^2$  are defined by

$$\text{SSE} = \sum_{i=1}^{20} \left( \frac{n_i}{N} - p_i(\hat{\theta}) \right)^2,$$

$$\text{SAE} = \sum_{i=1}^{20} \left| \frac{n_i}{N} - p_i(\hat{\theta}) \right|, \text{ and}$$

$$\chi^2 = N \sum_{i=1}^{20} \left[ \left( \frac{n_i}{N} - p_i(\hat{\theta}) \right)^2 / p_i(\hat{\theta}) \right],$$

where  $\hat{\theta}$  denotes the estimated parameter vector. The  $\chi^2$  is asymptotically distributed as a chi-square with degrees of freedom equal to the one less than the difference between the number of groups and the number of estimated parameters.

#### **D. The Gini Coefficient: a Measure of Inequality**

The Gini coefficient is widely used as a measure of income inequality. It has many desirable properties that make it convenient to compare inequality across countries and time. For example, the Gini coefficient is not affected by changes in the unit of measurement (currency), allowing researchers to avoid issues of inflation and purchasing power.

Conveniently, the Gini coefficient can be expressed mathematically in terms of the distributional parameters. These expressions can be found in McDonald (1984) for all the distributions considered except the generalized beta and the Dagum. Dagum (1977) demonstrated that the Gini coefficient for the Dagum distribution could be represented as

$$G_{DAGUM} = \frac{\Gamma(p)\Gamma(2p+1/a)}{\Gamma(p+1/a)\Gamma(2p)} - 1,$$

where  $\Gamma(\cdot)$  denotes the gamma function. This result also follows from the relationship between the Dagum, Singh-Maddala and GB2 distributions.

Using these expressions, Gini coefficients were calculated for each distribution in every data set considered, showing the effects of assuming different distributional models.

### **III. Description of the Income Data**

Household income data was obtained from the Luxembourg Income Study (LIS) database for 23 countries, including both developed and developing economies. European countries are perhaps over-represented within the LIS database because of its quality and availability. LIS data is grouped into 5-year waves, which, for a few countries, went back as far as the 1970's. In total, there were 82 data sets. Table 1 summarizes the countries and years that were examined.



Income Data Obtained From the Luxembourg Income Study					
	historical	wave I	wave II	wave III	wave IV
Australia		1981	1985	1989	1994
Belgium			1985	1988	1992,1997
Canada	1971,1975	1981	1987	1991	1994,1997
Czech Rep.				1992	1996
Denmark			1987	1992	1995,1997
Finland			1987	1991	1995
France		1979	1984	1989	1994
Germany	1973,1978	1981	1984	1989	1994
Hungary				1991	
Ireland				1987	
Israel		1979	1986	1992	1997
Italy			1986		1995
Mexico			1984	1989,1992	1994,1996
Netherlands			1983,1987	1991	1994
Norway		1979	1986	1991	1995
Poland			1986	1992	1995
Taiwan		1981	1986	1991	1995
Russia				1992	1995
Slovakia				1992	
Spain		1980		1990	
Sweden	1967,1975	1981	1987	1992	1995
Switzerland		1982		1992	
United States	1969,1974	1979	1986	1991	1994

Table 1

An advantage of using the LIS is that the data from each country is formatted as uniformly as possible, particularly concerning the definition of income, so that comparisons across countries and time are more plausible. For the purposes of this paper, we examine household data, defining income as gross wages and salaries, farm income, and any self-employment income. This definition of income seems appropriate since many of the distributions are based on models of pre-tax income generation. It is important to note that this is pre-tax household income, not including government transfer payments. Using different definitions of income can cause dramatic changes in the observed distribution of income.

Particularly, accounting for government redistribution will significantly alter the distribution of income in many of the countries considered.

In all cases income was measured in nominal local currency units. Because of government regulations and privacy laws, income data with individual observations are usually not available. In this analysis, the data was obtained in a grouped format with twenty equal probability intervals, corresponding to 5<sup>th</sup> through 95<sup>th</sup> percentiles.

#### **IV. Results**

Each distribution was fit to the 82 data sets, and goodness-of-fit criteria were calculated for each distribution, including the log-likelihood (Log-L) value as well as the SSE, SAE, and  $\chi^2$  value. The results for the United States 1997 are reported in table 2 and can be seen visually in figure 2.

The Weibull and Dagum are clearly the best fitting two and three parameter models, respectively. This holds true for either a SAE, SSE,  $\chi^2$ , or log-likelihood criteria. This also demonstrates the advantages of the three-parameter Dagum distribution over the two-parameter lognormal, gamma, and Weibull distributions. In this particular case, the four-parameter GB2 provides very marginal improvement relative to the Dagum. A comparison of the SSE, SAE, and  $\chi^2$  values show they are very similar for the Dagum and GB2 distributions. Examining their plots in figure 2 confirms they are nearly observationally equivalent. The same could said of the GB relative to the GB2.

### United States 1997

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Gini
gamma	-508.82	46,792	0.0010	0.1153	868	0.424
lognormal	-2844.48	52,892	0.0065	0.2917	6509	0.503
Weibull	-380.13	46,445	0.0007	0.1031	586	0.413
Dagum	-192.79	47,719	0.0003	0.0552	208	0.427
Beta 1	-440.62	46,372	0.0009	0.1133	706	0.414
Beta 2	-509.15	46,705	0.0010	0.1143	871	0.423
GG	-372.77	46,384	0.0007	0.1032	566	0.412
Singh-Maddala	-376.84	46,530	0.0007	0.1002	584	0.415
GB1	-372.77	46,384	0.0007	0.1032	566	0.412
GB2	-190.26	48,035	0.0003	0.0550	203	0.430
GB	-187.77	46,359	0.0003	0.0538	198	-

Table 2

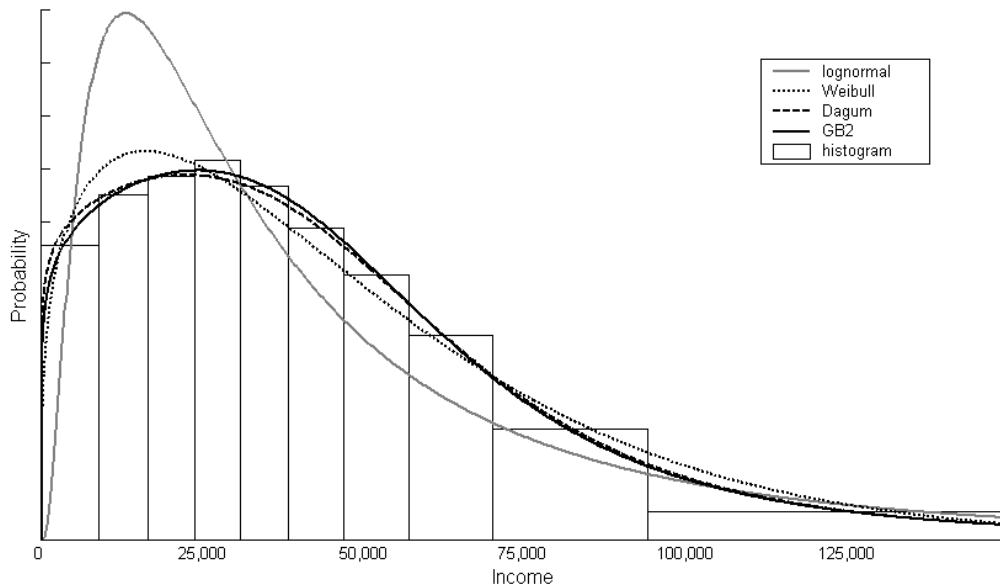


Figure 2

The nested relationship of the distributions guarantees the generalized distributions will fit the data at least as well as their special cases. However, this does not suggest their superiority

as a descriptive model will be statistically significant. Nested models can be compared using the likelihood ratio (LR) test:

$$LR = 2[\ln L(\theta_{ML}) - \ln L(\theta_R)],$$

where  $\theta_{ML}$  and  $\theta_R$  represent parameter estimates of the general and the restricted models, respectively. In cases in which the parameters are not on the boundary of the parameter spaces, the LR is asymptotically distributed as a  $\chi^2$  with degrees of freedom equal to the number of parameter restrictions imposed. This test cannot be used to compare non-nested models. Using this test at a one percent significance level to compare the fits obtained for the 1997 US data, the differences between the GB and GB2 and between the GB2 and Dagum are not significant. In this case, it is unclear if the improvements gained in adding parameters are of practical significance.

Detailed results for the other countries and time periods are found in the appendix. Table 3 summarizes these results by reporting the best fitting two-parameter, three-parameter, and four-parameter models for each of the 82 cases along with the corresponding log-likelihood values. Asterisks are placed next to log-likelihood values of the GB1, GB2 and GB distributions if adding the additional fourth or fifth parameter provided a statistically significant improvement, using the likelihood ratio test at the one percent level as outlined above.

Among the two-parameter models, the Weibull provides the best fit in sixty two percent of the cases. As in the case of the United States, introducing a third parameter provides a significantly improved fit. Among the three-parameter distributions, the Dagum is clearly the favorite. However, even though the Dagum distribution provides the best fit in 84 percent of the cases, it is not definitively the best of the three-parameter estimators in all cases. In fact, of the

**Best-Fitting Models**

country	year	two-parameter		three-parameter		four-parameter		five-parameter		Gini
		model	Log-L	model	Log-L	model	Log-L	model	Log-L	
<b>Australia</b>	1981	Weibull	-368.35	Dagum	-252.61	GB2	-247.5*	GB	-242.38	0.333
	1985	Weibull	-220.68	Dagum	-155.47	GB2	-150.78*	GB	-149.14	0.337
	1989	Weibull	-296.22	Dagum	-205.63	GB2	-202.41	GB	-202.41	0.349
	1994	Weibull	-204.52	Dagum	-153.58	GB2	-152.61	GB	-152.17	0.361
<b>Belgium</b>				0						
	1985	gamma	-235.41	GG	-227.34	GB1	-227.33	GB	-227.33	0.255
	1988	gamma	-202.09	GG	-200.79	GB1	-200.78	GB	-200.78	0.254
	1992	gamma	-83.68	GG	-82.59	GB2	-82.59	GB	-82.59	0.292
1997	gamma	-103.85	Dagum	-93.69	GB2	-92.47	GB	-91.69	0.327	
<b>Canada</b>	1971	Weibull	-611.61	Dagum	-192.48	GB2	-166.75*	GB	-166.14	0.379
	1975	Weibull	-481.71	Dagum	-171.46	GB2	-159.42*	GB	-156.27	0.372
	1981	Weibull	-273	Dagum	-117.24	GB2	-116.5	GB	-111.62*	0.372
	1987	Weibull	-251.37	Dagum	-104.27	GB2	-99.55*	GB	-97.3	0.386
	1991	Weibull	-286.14	Dagum	-105.53	GB2	-105.26	GB	-103.7	0.387
	1994	Weibull	-479.16	Dagum	-135.12	GB2	-132	GB	-127.17*	0.392
	1997	Weibull	-364.79	Dagum	-118.53	GB2	-117.9	GB	-116.52	0.392
<b>Czech Rep.</b>	1992	Weibull	-160.71	Dagum	-127.72	GB2	-121.39*	GB	-121.39	0.303
	1996	gamma	-125.71	Singh-Maddala	-101.44	GB2	-100.17	GB	-100.17	0.332
<b>Denmark</b>	1987	Weibull	-584.64	Dagum	-236.12	GB2	-233.65	GB	-233.65	0.386
	1992	Weibull	-710.57	Dagum	-277.62	GB2	-276.69	GB	-276.69	0.404
	1995	Weibull	-754.11	Dagum	-265.43	GB2	-257.38*	GB	-254.8	0.427
	1997	gamma	-1669.06	Dagum	-676.05	GB2	-664.5*	GB	-661.54	0.478
<b>Finland</b>	1987	Weibull	-555.6	Dagum	-289.9	GB2	-270.53*	GB	-258.62*	0.401
	1991	Weibull	-517.82	Dagum	-267.57	GB2	-244.33*	GB	-238.45*	0.417
	1995	Weibull	-772.69	Dagum	-341.4	GB2	-325.44*	GB	-322.37	0.433
<b>France</b>	1979	lognormal	-414.26	Dagum	-185.54	GB2	-133.5	GB	-133.5	0.504
	1984	Weibull	-193.93	Singh-Maddala	-169.2	GB2	-166.8	GB	-165.27	0.322
	1989	gamma	-141.78	Singh-Maddala	-117.92	GB2	-117.05	GB	-117.05	0.334
	1994	Weibull	-192.58	Dagum	-142.42	GB2	-141.8	GB	-141.79	0.387

Table 3

<b>Germany</b>	1973	Weibull	-4951.58	Dagum	-3502.02	GB2	-3303.37*	GB	-3291.03*	0.311
	1978	Weibull	-4555.74	Dagum	-3213.12	GB2	-3030.9*	GB	-3022.61*	0.319
	1981	gamma	-105.82	Dagum	-93.07	GB2	-93	GB	-92.4	0.274
	1984	gamma	-170.13	Dagum	-125.63	GB2	-125.34	GB	-124.78	0.299
	1989	Weibull	-125.56	Dagum	-105.87	GB2	-105.86	GB	-104.78	0.295
	1994	Weibull	-146.76	Dagum	-104.35	GB2	-99.84*	GB	-99.4	0.343
<b>Hungary</b>	1991	Weibull	-89.16	Dagum	-72.32	GB2	-70.13	GB	-69.86	0.428
<b>Ireland</b>	1987	Weibull	-96.5	Dagum	-84.06	GB2	-83.34	GB	-80.57	0.408
<b>Israel</b>	1979	Weibull	-69.97	Singh-Maddala	-69.26	GB2	-68.22	GB	-68.06	0.350
	1986	gamma	-76.01	Beta 2	-73.24	GB2	-72.86	GB	-72.83	0.388
	1992	gamma	-82.34	Singh-Maddala	-73.06	GB2	-73.02	GB	-72.55	0.405
	1997	gamma	-82.4	Dagum	-75.98	GB2	-74.37	GB	-73.97	0.430
<b>Italy</b>	1986	gamma	-244.65	Singh-Maddala	-233.17	GB2	-232.91	GB	-227.51*	0.306
	1995	gamma	-239.59	Dagum	-215.65	GB2	-214.55	GB	-209.14*	0.338
<b>Mexico</b>	1984	gamma	-122.13	Dagum	-90.73	GB2	-90.55	GB	-88.57	0.466
	1989	lognormal	-189.15	Singh-Maddala	-104.6	GB2	-105.59	GB	-104.59	0.494
	1992	lognormal	-245.56	Dagum	-107.74	GB2	-105.26	GB	-103.2	0.546
	1994	gamma	-357.1	Dagum	-115.67	GB2	-108.3*	GB	-104.67*	0.577
	1996	lognormal	-287.09	Dagum	-137.16	GB2	-133.91	GB	-128.21*	0.548
<b>Netherlands</b>	1983	gamma	-155.7	Dagum	-129.22	GB2	-128.59	GB	-123.89*	0.277
	1987	gamma	-134.19	Dagum	-113.59	GB2	-113.43	GB	-109.2*	0.288
	1991	Weibull	-356.09	Dagum	-248.86	GB2	-245.09*	GB	-243.69	0.326
	1994	Weibull	-369.98	Dagum	-230.55	GB2	-227.29	GB	-223.88*	0.340
<b>Norway</b>	1979	gamma	-944.48	Dagum	-625.37	GB2	-385.43*	GB	-385.43	0.528
	1986	Weibull	-372.86	Dagum	-219.58	GB2	-213.57*	GB	-210.84	0.343
	1991	Weibull	-409.31	Dagum	-231.58	GB2	-226.62*	GB	-225.21	0.362
	1995	Weibull	-554.06	Dagum	-258.06	GB2	-237.94*	GB	-231.54*	0.373

Table 3 cont.

<b>Poland</b>	1986	Weibull	-875.59	Dagum	-436.41	GB2	-421.29*	GB	-419.19	0.384
	1992	Weibull	-236.14	Dagum	-206.72	GB2	-206.71	GB	-204.95	0.358
	1995	gamma	-254.44	Beta 2	-148.55	GB2	-148.54	GB	-148.54	0.362
<b>Taiwan</b>	1981	lognormal	-227.66	Dagum	-164.88	GB2	-164.86	GB	-152.39*	0.293
	1986	gamma	-343.15	Dagum	-174.1	GB2	-164.45*	GB	-149.09*	0.314
	1991	gamma	-231.49	Dagum	-140.57	GB2	-139.96	GB	-114.96*	0.317
	1995	Weibull	-338.26	Dagum	-216.03	GB2	-211.46*	GB	-183.05*	0.341
<b>Russia</b>	1992	Weibull	-127.35	Dagum	-104.93	GB2	-104.93	GB	-104.01	0.466
	1995	gamma	-91.02	Dagum	-79.52	GB2	-79.05	GB	-78.93	0.537
<b>Slovakia</b>	1992	gamma	-140.79	GG	-137.54	GB1	-134.44	GB	-134.44	0.295
<b>Spain</b>	1980	gamma	-516.38	Dagum	-300.68	GB2	-272.21*	GB	-254.51*	0.356
	1990	gamma	-389.47	Dagum	-261.5	GB2	-258.35	GB	-242.13*	0.348
<b>Sweden</b>	1967	Weibull	-554.48	Dagum	-328.11	GB2	-293.38*	GB	-293.43	0.399
	1975	Weibull	-218.12	Dagum	-169.93	GB2	-169.98	GB	-144.6*	0.385
	1981	Weibull	-263.15	Dagum	-142.91	GB2	-142.67	GB	-138.33*	0.395
	1987	Weibull	-482.61	Dagum	-173.54	GB2	-166.15*	GB	-165.32	0.384
	1992	Weibull	-666.41	Dagum	-256.1	GB2	-245.12*	GB	-245.04	0.405
	1995	Weibull	-1179.37	Dagum	-424.18	GB2	-399.51*	GB	-398.88	0.446
<b>Switzerland</b>	1982	Weibull	-416.35	Dagum	-270.07	GB2	-204.79*	GB	-204.79	0.360
	1992	Weibull	-273.01	Dagum	-181.71	GB2	-156.25*	GB	-151.24*	0.373
<b>United States</b>	1969	Weibull	-321.88	Dagum	-170.59	GB2	-155.37*	GB	-154.32	0.368
	1974	Weibull	-264.27	Dagum	-144.99	GB2	-135.84*	GB	-134.17	0.379
	1979	Weibull	-251.07	Dagum	-154.41	GB2	-148.44*	GB	-146.57	0.392
	1986	Weibull	-186.99	Dagum	-109.58	GB2	-103.19*	GB	-101.34	0.408
	1991	Weibull	-206.76	Dagum	-137.27	GB2	-136.66	GB	-133.86	0.417
	1994	Weibull	-443.19	Dagum	-229.76	GB2	-228.3	GB	-226.1	0.425
	1997	Weibull	-380.13	Dagum	-192.79	GB2	-190.26	GB	-187.77	0.430

Table 3 cont.

three-parameter distributions, all but the beta 1 outperform the others in certain cases. Within a given country, the relative ordering of the models in terms of their goodness-of-fit generally holds constant over time, but this is not always the case. For example, the distribution of income in Mexico (see table 3) is best modeled by the Dagum in 1984, 1992, 1994, and 1996, but in 1989, the Singh-Maddala distribution provides a better fit as measured by log-likelihood, SAE, or  $\chi^2$  value. The GB2 is clearly the best four-parameter distribution. Its additional parameter makes it significantly better than the best three-parameter distribution in 44% of the data sets considered.

In almost every case, the four measures of goodness-of-fit (log-likelihood, SSE, SAE, and  $\chi^2$ ) agree in the relative ranking of the models considered. In the few cases where they do not agree, the difference is negligible. In Australia 1989, for example, the GB has a log-likelihood value just barely better than that of the GB2 (by a difference of 0.15) but has a SAE value slightly worse than that of the GB2 (by a difference of 0.007).

In nearly all of the cases considered, the generalized beta family of distributions has the flexibility to provide a good fit to the data. This can be seen by a visual inspection of the histogram overlaid with the fitted pdf, an inspection of the SSE, SAE, or  $\chi^2$  values, or a comparison of the sample mean with the expected value of the fitted pdf. In a few cases, no model provided a good fit. This is most obvious in the Scandinavian countries of Finland, Norway, and Sweden where the data shows a large clump of people earning very low incomes, giving the observed distribution a spike at the lowest income levels, which causes the sample mean to be significantly lower than the fitted (unimodal) models predict.

The choice of functional model strongly affects the predicted level of inequality as measured by the Gini coefficient. Generally, the Gini coefficients estimated by the lognormal



are the highest, the Weibull and generalized gamma are the lowest, and the Dagum and GB2 find themselves somewhere in between. The disagreement results from inferior fits, particularly in the tails. It is also interesting that in almost every country, the Gini coefficients increase monotonically over time. It is important to note that these show inequality in income earned before transfer payments. Countries with progressive tax structures and large social programs and transfer payments may not observe such inequality after income redistribution—but looking at pre-tax earnings there is a general tendency towards greater inequality.

## **V. Conclusions**

This paper compares the ability of eleven probability distribution functions to fit income data for twenty three countries over time; in total, eighty two data sets were analyzed. Given the estimated distributional parameters, Gini coefficients were calculated for each country. Examining these Gini coefficients over multiple time periods reveals a general trend of increasing inequality for almost all the countries considered. Concerning functional form, the Weibull distribution is the best fitting two-parameter distribution in 62 percent of the data sets, the Dagum is the best fitting three-parameter distribution in 84 percent of the time, and the GB2 provides a statistically significant (at the one percent level) improvement over all the three-parameter distributions in 44 percent of the cases considered.

Appendix

Australia 1981

sample	
mean	obs
21,338	10568

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-600.31	21,160	0.0052	0.2287	1164	8740	2.421	-	-	-	0.344
lognormal	-1440.09	22,757	0.0134	0.3817	3718	9.758	0.742	-	-	-	0.400
Weibull	-368.35	20,889	0.0029	0.1631	617	1.777	23475	-	-	-	0.323
Dagum	-252.61	21,018	0.0018	0.1297	350	4.873	30808	0.296	-	-	0.328
Beta 1	-406.18	20,792	0.0035	0.2020	657	67716	1.798	4.058	-	-	0.324
Beta 2	-600.42	21,188	0.0052	0.2308	1156	2E+11	2.407	2E+07	-	-	0.345
GG	-326.68	20,806	0.0026	0.1772	493	2.383	30284	0.635	-	-	0.322
Singh-Maddala	-368.35	20,889	0.0029	0.1631	617	1.777	3E+07	3E+05	-	-	0.323
GB1	-326.68	20,806	0.0026	0.1771	493	2.383	4E+06	0.635	84327	-	0.322
GB2	-247.50	21,183	0.0018	0.1300	340	6.223	28340	0.228	0.6506	-	0.333
GB	-242.38	20,630	0.0017	0.1211	330	7.71	26097	0.992	0.1838	0.422	-

Australia 1985

sample	
mean	obs
29,439	5492

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-339.70	29,145	0.0050	0.2266	613	12260	2.377	-	-	-	0.347
lognormal	-784.97	31,432	0.0130	0.3766	2020	10.07	0.751	-	-	-	0.404
Weibull	-220.68	28,768	0.0028	0.1648	324	1.758	32309	-	-	-	0.326
Dagum	-155.47	28,929	0.0017	0.1236	170	4.83	42482	0.296	-	-	0.331
Beta 1	-242.51	28,637	0.0035	0.2049	346	95573	1.783	4.169	-	-	0.327
Beta 2	-339.72	29,166	0.0050	0.2278	611	2E+11	2.37	2E+07	-	-	0.348
GG	-199.16	28,650	0.0026	0.1781	255	2.349	41698	0.638	-	-	0.325
Singh-Maddala	-220.68	28,768	0.0028	0.1648	324	1.758	4E+07	3E+05	-	-	0.326
GB1	-199.16	28,650	0.0026	0.1781	255	2.349	6E+06	0.639	105117	-	0.325
GB2	-150.78	29,236	0.0016	0.1190	160	6.74	38276	0.208	0.5687	-	0.337
GB	-149.14	28,438	0.0016	0.1148	157	7.991	36104	0.995	0.1749	0.41	-

Australia 1989

sample	
mean	obs
40,755	10629

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-454.81	39,755	0.0035	0.2002	835	17718	2.244	-	-	-	0.356
lognormal	-1209.18	42,868	0.0101	0.3479	3044	10.37	0.768	-	-	-	0.413
Weibull	-296.22	39,302	0.0021	0.1437	457	1.677	44006	-	-	-	0.339
Dagum	-205.63	39,726	0.0013	0.1099	262	4.4	57344	0.318	-	-	0.345
Beta 1	-339.14	39,175	0.0026	0.1709	529	2E+05	1.776	5.266	-	-	0.339
Beta 2	-456.43	39,543	0.0034	0.1948	864	3E+11	2.296	2E+07	-	-	0.353
GG	-279.08	39,199	0.0020	0.1528	405	2.035	53445	0.738	-	-	0.337
Singh-Maddala	-296.22	39,302	0.0021	0.1437	457	1.677	4E+07	1E+05	-	-	0.339
GB1	-279.08	39,200	0.0020	0.1528	405	2.035	2E+07	0.738	154710	-	0.337
GB2	-202.41	40,012	0.0012	0.1083	256	5.317	53464	0.259	0.7172	-	0.349
GB	-202.26	39,382	0.0012	0.1090	256	5.52	52166	0.994	0.2495	0.649	-

### Australia 1994

<i>sample</i>	
<i>mean</i>	<i>obs</i>
45,825	4619

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-289.04	44,760	0.0051	0.2216	487	22379	2	-	-	-	0.375
lognormal	-693.82	49,471	0.0141	0.3783	1748	10.46	0.834	-	-	-	0.445
Weibull	-204.52	44,175	0.0031	0.1560	292	1.592	49247	-	-	-	0.353
Dagum	-153.58	44,513	0.0021	0.1399	167	4.575	68511	0.275	-	-	0.358
Beta 1	-210.53	43,884	0.0036	0.1909	280	2E+05	1.536	3.856	-	-	0.352
Beta 2	-289.05	44,793	0.0051	0.2223	485	4E+11	1.995	2E+07	-	-	0.375
GG	-178.95	43,939	0.0028	0.1747	216	2.267	68116	0.583	-	-	0.350
Singh-Maddala	-204.52	44,176	0.0031	0.1561	292	1.592	2E+08	4E+05	-	-	0.353
GB1	-178.95	43,939	0.0028	0.1748	216	2.267	1E+07	0.583	97488	-	0.350
GB2	-152.61	44,793	0.0021	0.1393	165	5.436	64046	0.229	0.7336	-	0.361
GB	-152.17	43,815	0.0021	0.1379	165	6.094	60014	0.988	0.2039	0.557	-

### Belgium 1985

<i>sample</i>	
<i>mean</i>	<i>obs</i>
6,494	4357

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-235.41	6,484	0.0037	0.2229	366	1357	4.778	-	-	-	0.251
lognormal	-236.42	6,556	0.0037	0.2228	362	8.676	0.473	-	-	-	0.262
Weibull	-322.43	6,452	0.0057	0.2809	563	2.348	7280	-	-	-	0.256
Dagum	-271.29	6,671	0.0047	0.2516	434	3.823	6374	0.83	-	-	0.275
Beta 1	-235.41	6,484	0.0037	0.2229	366	3E+09	4.777	3E+06	-	-	0.251
Beta 2	-229.42	6,511	0.0036	0.2145	349	21680	6.061	21.18	-	-	0.256
GG	-227.34	6,510	0.0035	0.2123	345	0.507	21.05	17.79	-	-	0.255
Singh-Maddala	-255.78	6,544	0.0042	0.2369	406	3.078	7581	1.745	-	-	0.262
GB1	-227.33	6,510	0.0035	0.2123	345	0.503	6E+08	17.99	5749.5	-	0.255
GB2	-227.39	6,510	0.0035	0.2123	345	0.523	1E+07	16.96	980.12	-	0.255
GB	-227.33	6,510	0.0035	0.2123	345	0.503	6E+08	0	17.986	5755	-

### Belgium 1988

<i>sample</i>	
<i>mean</i>	<i>obs</i>
6,942	2518

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-202.09	7,000	0.0049	0.2590	324	1466	4.774	-	-	-	0.252
lognormal	-211.24	7,083	0.0054	0.2621	338	8.753	0.475	-	-	-	0.263
Weibull	-240.92	6,964	0.0064	0.3056	415	2.37	7858	-	-	-	0.254
Dagum	-230.17	7,166	0.0059	0.2831	391	4.159	7491	0.675	-	-	0.270
Beta 1	-202.09	7,001	0.0049	0.2590	324	1E+10	4.771	8E+06	-	-	0.252
Beta 2	-201.61	7,011	0.0049	0.2584	322	74124	5.181	55.77	-	-	0.253
GG	-200.79	7,015	0.0049	0.2569	320	0.732	357.1	8.657	-	-	0.254
Singh-Maddala	-216.19	7,034	0.0054	0.2745	359	2.916	9366	2.301	-	-	0.258
GB1	-200.49	7,008	0.0048	0.2571	320	0.678	2E+05	9.144	89.501	-	0.253
GB2	-200.79	7,014	0.0049	0.2569	320	0.736	2E+08	8.56	15394	-	0.254
GB	-200.78	7,014	0.0049	0.2569	320	0.731	5E+07	0	8.6649	5657	-

### Belgium 1992

<i>sample</i>	
<i>mean</i>	<i>obs</i>
12,231	2581

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-83.68	12,761	0.0008	0.1075	44	3601	3.544	-	-	-	0.289
lognormal	-103.02	13,012	0.0016	0.1365	84	9.317	0.56	-	-	-	0.308
Weibull	-118.30	12,696	0.0022	0.1663	114	2.021	14328	-	-	-	0.290
Dagum	-93.05	13,081	0.0012	0.1296	63	3.755	14112	0.611	-	-	0.307
Beta 1	-83.68	12,761	0.0008	0.1076	44	4E+10	3.543	1E+07	-	-	0.289
Beta 2	-82.61	12,793	0.0008	0.0974	42	1E+05	3.883	36.23	-	-	0.292
GG	-82.59	12,787	0.0008	0.0972	42	0.805	1560	5.315	-	-	0.292
Singh-Maddala	-85.95	12,876	0.0009	0.1066	49	2.532	17010	2.171	-	-	0.297
GB1	-82.59	12,787	0.0008	0.0972	42	0.803	7E+07	5.329	5558.4	-	0.292
GB2	-82.59	12,788	0.0008	0.0972	42	0.843	1E+06	4.97	209.85	-	0.292
GB	-82.59	12,788	0.0008	0.0972	42	0.844	1E+06	1	4.9604	203.9	-

### Belgium 1997

<i>sample</i>	
<i>mean</i>	<i>obs</i>
1,580,581	2906

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-103.85	1,652,354	0.0015	0.1224	81	6E+05	2.789	-	-	-	0.323
lognormal	-185.57	1,713,748	0.0040	0.2065	276	14.14	0.651	-	-	-	0.355
Weibull	-110.23	1,641,387	0.0016	0.1464	95	1.813	2E+06	-	-	-	0.318
Dagum	-93.69	1,682,970	0.0010	0.1149	62	3.776	2E+06	0.479	-	-	0.333
Beta 1	-102.29	1,647,344	0.0014	0.1277	77	2E+07	2.587	27.99	-	-	0.320
Beta 2	-103.89	1,650,318	0.0015	0.1211	81	1E+13	2.807	2E+07	-	-	0.322
GG	-100.38	1,646,775	0.0013	0.1277	74	1.285	1E+06	1.801	-	-	0.319
Singh-Maddala	-94.61	1,657,103	0.0011	0.1077	63	2.089	3E+06	3.534	-	-	0.323
GB1	-100.38	1,646,787	0.0013	0.1277	74	1.285	2E+10	1.801	224989	-	0.319
GB2	-92.47	1,667,672	0.0010	0.1077	59	2.999	2E+06	0.629	1.5555	-	0.327
GB	-91.69	1,638,410	0.0010	0.1044	58	3.661	2E+06	0.967	0.5075	0.842	-

### Canada 1971

<i>sample</i>	
<i>mean</i>	<i>obs</i>
8,633	20386

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-1004.16	8,224	0.0044	0.2480	2002	4805	1.711	-	-	-	0.401
lognormal	-3039.14	9,420	0.0148	0.4304	7442	8.721	0.927	-	-	-	0.488
Weibull	-611.61	8,104	0.0023	0.1746	1193	1.474	8957	-	-	-	0.375
Dagum	-192.48	8,098	0.0005	0.0818	223	5.099	13799	0.214	-	-	0.371
Beta 1	-466.35	8,002	0.0020	0.1636	769	25410	1.285	2.796	-	-	0.369
Beta 2	-1004.16	8,223	0.0044	0.2479	2003	4E+10	1.712	8E+06	-	-	0.401
GG	-306.15	8,021	0.0011	0.1266	446	2.694	14478	0.417	-	-	0.367
Singh-Maddala	-611.62	8,104	0.0023	0.1746	1193	1.474	7E+07	6E+05	-	-	0.375
GB1	-306.15	8,021	0.0011	0.1266	446	2.694	1E+06	0.417	106495	-	0.366
GB2	-166.75	8,222	0.0004	0.0669	171	8.654	11927	0.123	0.4281	-	0.379
GB	-166.14	8,010	0.0004	0.0656	169	9.409	11664	0.998	0.1134	0.37	-

### Canada 1975

<i>sample</i>	
<i>mean</i>	<i>obs</i>
14,052	21750

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-866.89	13,370	0.0035	0.2193	1698	7310	1.829	-	-	-	0.390
lognormal	-2795.66	14,993	0.0126	0.4030	6774	9.227	0.881	-	-	-	0.467
Weibull	-481.71	13,192	0.0016	0.1413	884	1.519	14634	-	-	-	0.366
Dagum	-171.46	13,252	0.0004	0.0676	178	4.8	21578	0.241	-	-	0.367
Beta 1	-422.94	13,066	0.0016	0.1488	682	43902	1.392	3.286	-	-	0.363
Beta 2	-867.03	13,388	0.0035	0.2202	1690	6E+10	1.821	9E+06	-	-	0.391
GG	-281.75	13,097	0.0009	0.1147	396	2.439	22031	0.495	-	-	0.361
Singh-Maddala	-481.71	13,192	0.0016	0.1413	884	1.519	8E+07	5E+05	-	-	0.366
GB1	-281.76	13,097	0.0009	0.1147	396	2.44	2E+06	0.494	91916	-	0.361
GB2	-159.42	13,393	0.0004	0.0621	153	6.615	19359	0.172	0.5782	-	0.372
GB	-156.27	13,027	0.0003	0.0589	147	7.839	18215	0.994	0.1448	0.418	-

### Canada 1981

<i>sample</i>	
<i>mean</i>	<i>obs</i>
24,969	12331

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-477.91	23,898	0.0032	0.2049	853	13331	1.793	-	-	-	0.393
lognormal	-1536.26	26,818	0.0122	0.3948	3520	9.802	0.889	-	-	-	0.470
Weibull	-273.00	23,589	0.0014	0.1302	428	1.498	26127	-	-	-	0.370
Dagum	-117.24	23,740	0.0003	0.0661	81	4.828	39195	0.234	-	-	0.370
Beta 1	-218.98	23,349	0.0012	0.1285	287	76836	1.353	3.1	-	-	0.365
Beta 2	-477.92	23,888	0.0032	0.2048	854	2E+11	1.795	2E+07	-	-	0.393
GG	-155.43	23,422	0.0006	0.0976	158	2.471	40078	0.478	-	-	0.364
Singh-Maddala	-273.01	23,589	0.0014	0.1302	428	1.498	2E+08	5E+05	-	-	0.370
GB1	-155.43	23,423	0.0006	0.0976	158	2.471	5E+06	0.478	122069	-	0.364
GB2	-116.50	23,826	0.0003	0.0667	79	5.373	37435	0.209	0.8175	-	0.372
GB	-111.62	23,232	0.0003	0.0610	69	7.315	32558	0.987	0.1527	0.427	-

### Canada 1987

<i>sample</i>	
<i>mean</i>	<i>obs</i>
38,275	8667

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-390.95	33,944	0.0035	0.2187	683	20641	1.644	-	-	-	0.408
lognormal	-1211.14	38,991	0.0133	0.4094	2811	10.13	0.944	-	-	-	0.495
Weibull	-251.37	33,485	0.0018	0.1504	396	1.43	36856	-	-	-	0.384
Dagum	-104.27	33,577	0.0004	0.0714	61	4.866	57426	0.218	-	-	0.381
Beta 1	-193.71	33,068	0.0014	0.1371	242	1E+05	1.253	2.909	-	-	0.377
Beta 2	-390.97	33,916	0.0035	0.2184	685	1E+11	1.649	7E+06	-	-	0.408
GG	-140.44	33,172	0.0008	0.1083	134	2.537	59747	0.434	-	-	0.375
Singh-Maddala	-251.37	33,485	0.0018	0.1504	396	1.43	4E+08	7E+05	-	-	0.384
GB1	-140.44	33,173	0.0008	0.1083	134	2.537	6E+06	0.434	98095	-	0.375
GB2	-99.55	33,968	0.0003	0.0636	52	6.946	50974	0.15	0.5467	-	0.386
GB	-97.30	32,903	0.0003	0.0589	47	9.189	47134	0.995	0.1131	0.339	-

### Canada 1991

<i>sample</i>	
<i>mean</i>	<i>obs</i>
45,589	15257

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-489.38	41,563	0.0027	0.1887	852	25145	1.653	-	-	-	0.407
lognormal	-1758.46	47,208	0.0114	0.3818	3891	10.33	0.931	-	-	-	0.489
Weibull	-286.14	41,060	0.0013	0.1281	438	1.421	45153	-	-	-	0.386
Dagum	-105.53	41,377	0.0002	0.0459	53	4.63	69595	0.231	-	-	0.385
Beta 1	-218.66	40,610	0.0010	0.1157	276	1E+05	1.284	3.269	-	-	0.380
Beta 2	-489.39	41,563	0.0027	0.1887	852	6E+09	1.653	2E+05	-	-	0.407
GG	-151.52	40,758	0.0005	0.0826	143	2.325	70577	0.484	-	-	0.378
Singh-Maddala	-286.14	41,060	0.0013	0.1281	438	1.421	5E+08	6E+05	-	-	0.386
GB1	-151.52	40,759	0.0005	0.0826	143	2.325	1E+07	0.484	89571	-	0.378
GB2	-104.26	41,576	0.0002	0.0453	50	5.294	65805	0.2	0.7831	-	0.387
GB	-103.70	40,779	0.0002	0.0430	49	5.918	61515	0.985	0.1787	0.594	-

### Canada 1994

<i>sample</i>	
<i>mean</i>	<i>obs</i>
47,549	28155

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-839.48	45,218	0.0026	0.1903	1578	28132	1.607	-	-	-	0.412
lognormal	-3240.70	51,732	0.0116	0.3927	7391	10.4	0.948	-	-	-	0.497
Weibull	-479.16	44,671	0.0013	0.1267	844	1.397	48998	-	-	-	0.391
Dagum	-135.12	45,010	0.0002	0.0472	101	4.535	75904	0.232	-	-	0.390
Beta 1	-352.06	44,166	0.0010	0.1224	535	2E+05	1.257	3.285	-	-	0.384
Beta 2	-840.17	45,356	0.0027	0.1924	1569	4E+11	1.594	1E+07	-	-	0.414
GG	-224.78	44,324	0.0005	0.0883	280	2.292	77266	0.482	-	-	0.383
Singh-Maddala	-479.16	44,672	0.0013	0.1267	844	1.397	7E+08	6E+05	-	-	0.391
GB1	-224.79	44,324	0.0005	0.0884	280	2.291	1E+07	0.482	97755	-	0.383
GB2	-132.00	45,270	0.0002	0.0448	95	5.281	71142	0.198	0.7575	-	0.392
GB	-127.17	44,069	0.0002	0.0391	85	6.588	63832	0.987	0.1577	0.471	-

### Canada 1997

<i>sample</i>	
<i>mean</i>	<i>obs</i>
50,742	25018

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-659.59	47,738	0.0022	0.1775	1225	29413	1.623	-	-	-	0.410
lognormal	-2700.64	54,314	0.0104	0.3737	6281	10.46	0.939	-	-	-	0.493
Weibull	-364.79	47,187	0.0010	0.1151	612	1.399	51769	-	-	-	0.391
Dagum	-118.53	47,665	0.0001	0.0376	69	4.395	79112	0.243	-	-	0.391
Beta 1	-289.08	46,706	0.0008	0.1060	412	2E+05	1.285	3.578	-	-	0.385
Beta 2	-659.71	47,802	0.0022	0.1782	1221	7E+11	1.617	2E+07	-	-	0.411
GG	-188.87	46,867	0.0004	0.0768	210	2.17	79031	0.52	-	-	0.383
Singh-Maddala	-364.79	47,187	0.0010	0.1151	612	1.399	7E+08	6E+05	-	-	0.391
GB1	-188.87	46,867	0.0004	0.0768	210	2.17	2E+07	0.52	90672	-	0.383
GB2	-117.90	47,800	0.0001	0.0376	68	4.724	76517	0.225	0.8738	-	0.392
GB	-116.52	46,824	0.0001	0.0371	65	5.402	69783	0.981	0.1958	0.618	-

### Czech Rep. 1992

<i>sample</i>	
<i>mean</i>	<i>obs</i>
964	11512

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-163.79	944	0.0008	0.1051	176	298.2	3.167	-	-	-	0.305
lognormal	-470.84	973	0.0033	0.2220	847	6.696	0.607	-	-	-	0.332
Weibull	-160.71	938	0.0007	0.1100	170	1.958	1058	-	-	-	0.298
Dagum	-127.72	955	0.0004	0.0702	104	4.373	1214	0.419	-	-	0.308
Beta 1	-142.34	940	0.0006	0.0957	131	6041	2.738	14.87	-	-	0.299
Beta 2	-163.79	944	0.0008	0.1050	176	3E+09	3.171	9E+06	-	-	0.305
GG	-135.13	940	0.0005	0.0909	117	1.468	719.4	1.626	-	-	0.299
Singh-Maddala	-125.42	943	0.0004	0.0803	98	2.17	2052	4.872	-	-	0.301
GB1	-135.14	940	0.0005	0.0909	117	1.468	1E+06	1.626	49023	-	0.299
GB2	-121.39	947	0.0004	0.0698	90	3.046	1440	0.647	2.0118	-	0.303
GB	-121.39	947	0.0004	0.0696	90	3.068	1432	1	0.6417	1.979	-

### Czech Rep. 1996

<i>sample</i>	
<i>mean</i>	<i>obs</i>
2,168	19442

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-125.71	2,042	0.0002	0.0551	88	752.3	2.714	-	-	-	0.327
lognormal	-497.53	2,113	0.0020	0.1677	888	7.441	0.656	-	-	-	0.357
Weibull	-248.77	2,031	0.0009	0.1065	337	1.771	2282	-	-	-	0.324
Dagum	-117.56	2,091	0.0002	0.0496	74	3.596	2449	0.506	-	-	0.341
Beta 1	-125.71	2,042	0.0002	0.0551	88	3E+06	2.713	3469	-	-	0.327
Beta 2	-125.71	2,042	0.0002	0.0551	88	6E+08	2.715	8E+05	-	-	0.327
GG	-124.88	2,041	0.0002	0.0545	86	1.054	873.2	2.471	-	-	0.326
Singh-Maddala	-101.44	2,056	0.0001	0.0385	40	2.096	3407	2.959	-	-	0.331
GB1	-124.88	2,041	0.0002	0.0545	86	1.054	3E+07	2.471	63631	-	0.326
GB2	-100.17	2,060	0.0001	0.0360	38	2.391	3042	0.839	2.2023	-	0.332
GB	-100.17	2,061	0.0001	0.0359	38	2.394	3040	1	0.8377	2.196	-

### Denmark 1987

<i>sample</i>	
<i>mean</i>	<i>obs</i>
222,676	9111

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-778.20	224,395	0.0080	0.3228	1549	2E+05	1.467	-	-	-	0.428
lognormal	-2004.38	276,078	0.0235	0.5464	5192	11.98	1.051	-	-	-	0.543
Weibull	-584.64	220,447	0.0050	0.2482	1232	1.374	2E+05	-	-	-	0.396
Dagum	-236.12	218,040	0.0017	0.1464	335	6.256	4E+05	0.151	-	-	0.383
Beta 1	-332.18	215,681	0.0029	0.1938	524	6E+05	1.05	1.821	-	-	0.381
Beta 2	-778.21	224,547	0.0080	0.3232	1546	1E+12	1.465	9E+06	-	-	0.429
GG	-256.58	216,146	0.0020	0.1599	374	3.641	4E+05	0.264	-	-	0.379
Singh-Maddala	-584.64	220,450	0.0050	0.2481	1232	1.374	5E+09	8E+05	-	-	0.396
GB1	-256.58	216,149	0.0020	0.1599	374	3.641	9E+06	0.264	50415	-	0.379
GB2	-233.65	219,808	0.0017	0.1411	332	8.873	4E+05	0.105	0.5472	-	0.386
GB	-233.65	219,779	0.0017	0.1411	332	8.842	4E+05	1	0.1058	0.55	-

### Denmark 1992

<i>sample</i>	
<i>mean</i>	<i>obs</i>
258,142	9104

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-854.99	261,240	0.0090	0.3454	1677	2E+05	1.277	-	-	-	0.454
lognormal	-2190.64	342,297	0.0263	0.5792	5485	12.07	1.16	-	-	-	0.588
Weibull	-710.57	256,863	0.0065	0.2970	1496	1.263	3E+05	-	-	-	0.422
Dagum	-277.62	252,287	0.0021	0.1662	433	6.62	5E+05	0.129	-	-	0.402
Beta 1	-354.88	249,591	0.0030	0.2031	587	7E+05	0.92	1.597	-	-	0.400
Beta 2	-855.21	261,233	0.0090	0.3455	1678	1E+09	1.277	7084	-	-	0.454
GG	-289.38	250,075	0.0022	0.1789	455	3.995	6E+05	0.216	-	-	0.398
Singh-Maddala	-710.57	256,865	0.0065	0.2970	1496	1.263	1E+10	9E+05	-	-	0.422
GB1	-289.38	250,075	0.0022	0.1789	455	3.994	9E+06	0.216	79005	-	0.398
GB2	-276.69	253,780	0.0021	0.1606	434	8.537	5E+05	0.099	0.625	-	0.404
GB	-276.69	253,775	0.0021	0.1606	434	8.537	5E+05	1	0.0991	0.625	-

### Denmark 1995

<i>sample</i>	
<i>mean</i>	<i>obs</i>
279,396	9289

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-850.05	281,377	0.0090	0.3572	1623	2E+05	1.144	-	-	-	0.474
lognormal	-2270.97	391,087	0.0272	0.5915	5445	12.09	1.252	-	-	-	0.624
Weibull	-754.11	277,268	0.0072	0.3103	1548	1.175	3E+05	-	-	-	0.446
Dagum	-265.43	269,886	0.0021	0.1632	391	6.073	6E+05	0.132	-	-	0.420
Beta 1	-371.18	267,629	0.0033	0.2128	601	8E+05	0.854	1.621	-	-	0.419
Beta 2	-850.66	283,252	0.0091	0.3584	1611	3E+12	1.129	1E+07	-	-	0.477
GG	-289.48	267,853	0.0023	0.1794	439	3.76	6E+05	0.214	-	-	0.416
Singh-Maddala	-754.11	277,272	0.0072	0.3103	1548	1.175	5E+10	1E+06	-	-	0.446
GB1	-289.48	267,854	0.0023	0.1794	439	3.76	1E+07	0.214	102065	-	0.416
GB2	-257.38	274,268	0.0019	0.1536	378	12.12	5E+05	0.065	0.3316	-	0.427
GB	-254.80	264,523	0.0019	0.1461	374	23.99	4E+05	1	0.0328	0.139	-

### Denmark 1997

<i>sample</i>	
<i>mean</i>	<i>obs</i>
286,881	9941

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-1669.06	292,836	0.0172	0.4953	3693	4E+05	0.771	-	-	-	0.551
lognormal	-4280.06	754,740	0.0496	0.7872	13436	11.89	1.815	-	-	-	0.801
Weibull	-1838.07	294,664	0.0166	0.4877	5444	0.921	3E+05	-	-	-	0.529
Dagum	-676.05	262,846	0.0060	0.2716	1316	7.809	7E+05	0.078	-	-	0.470
Beta 1	-763.71	266,913	0.0070	0.3144	1474	8E+05	0.59	1.07	-	-	0.466
Beta 2	-1669.06	292,803	0.0172	0.4953	3693	4E+12	0.771	1E+07	-	-	0.551
GG	-689.90	261,964	0.0061	0.2822	1341	5.644	7E+05	0.107	-	-	0.467
Singh-Maddala	-1838.07	294,656	0.0166	0.4877	5443	0.921	9E+11	1E+06	-	-	0.529
GB1	-689.90	261,965	0.0061	0.2822	1341	5.644	5E+06	0.107	77555	-	0.467
GB2	-664.50	267,151	0.0059	0.2620	1292	19.69	5E+05	0.031	0.217	-	0.478
GB	-661.54	252,964	0.0058	0.2542	1287	87.45	5E+05	1	0.007	0.039	-



## Finland 1987

<i>sample</i>	
<i>mean</i>	<i>obs</i>
116,212	10423

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-755.54	145,143	0.0063	0.2593	1608	94706	1.533	-	-	-	0.421
lognormal	-2088.12	175,162	0.0198	0.4722	6259	11.56	1.016	-	-	-	0.528
Weibull	-555.60	142,789	0.0039	0.1931	1226	1.391	2E+05	-	-	-	0.392
Dagum	-289.90	142,065	0.0022	0.1480	436	4.703	2E+05	0.22	-	-	0.389
Beta 1	-468.97	140,695	0.0040	0.2223	811	5E+05	1.177	2.717	-	-	0.386
Beta 2	-755.81	145,608	0.0064	0.2616	1588	3E+12	1.519	3E+07	-	-	0.422
GG	-361.07	140,765	0.0029	0.1920	577	2.585	3E+05	0.406	-	-	0.383
Singh-Maddala	-555.60	142,791	0.0039	0.1930	1226	1.391	2E+09	6E+05	-	-	0.392
GB1	-361.07	140,766	0.0029	0.1920	577	2.585	2E+07	0.406	65960	-	0.383
GB2	-270.53	145,194	0.0020	0.1385	394	8.863	2E+05	0.115	0.364	-	0.401
GB	-258.62	138,195	0.0019	0.1231	370	14.05	2E+05	1	0.0721	0.192	-

## Finland 1991

<i>sample</i>	
<i>mean</i>	<i>obs</i>
149,614	10160

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-672.53	180,148	0.0055	0.2552	1391	1E+05	1.423	-	-	-	0.434
lognormal	-1963.75	221,851	0.0186	0.4679	5740	11.75	1.063	-	-	-	0.548
Weibull	-517.82	177,451	0.0037	0.1989	1108	1.322	2E+05	-	-	-	0.408
Dagum	-267.57	176,644	0.0019	0.1431	394	4.452	3E+05	0.221	-	-	0.404
Beta 1	-440.93	174,718	0.0037	0.2226	754	6E+05	1.125	2.924	-	-	0.401
Beta 2	-671.40	178,791	0.0054	0.2473	1429	2E+13	1.451	1E+08	-	-	0.430
GG	-341.76	174,832	0.0027	0.1900	540	2.408	3E+05	0.417	-	-	0.397
Singh-Maddala	-517.82	177,454	0.0037	0.1989	1108	1.322	4E+09	5E+05	-	-	0.408
GB1	-341.76	174,832	0.0027	0.1900	540	2.408	5E+07	0.417	148398	-	0.397
GB2	-244.33	181,483	0.0017	0.1326	345	9.59	3E+05	0.1	0.3136	-	0.417
GB	-238.45	171,736	0.0017	0.1235	332	13.97	2E+05	1	0.0686	0.19	-

## Finland 1995

<i>sample</i>	
<i>mean</i>	<i>obs</i>
143,892	7913

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-867.08	177,156	0.0105	0.3625	1844	2E+05	1.142	-	-	-	0.475
lognormal	-2298.47	258,059	0.0311	0.6241	7050	11.63	1.287	-	-	-	0.637
Weibull	-772.69	174,225	0.0079	0.2914	1957	1.185	2E+05	-	-	-	0.443
Dagum	-341.40	168,830	0.0035	0.2064	558	5.473	3E+05	0.149	-	-	0.422
Beta 1	-475.14	168,144	0.0054	0.2802	828	5E+05	0.869	1.745	-	-	0.421
Beta 2	-867.09	177,346	0.0105	0.3631	1839	2E+12	1.14	1E+07	-	-	0.475
GG	-377.53	167,760	0.0040	0.2372	626	3.373	4E+05	0.242	-	-	0.418
Singh-Maddala	-772.69	174,232	0.0079	0.2914	1957	1.185	2E+10	8E+05	-	-	0.443
GB1	-377.53	167,760	0.0040	0.2372	626	3.373	1E+07	0.242	96676	-	0.418
GB2	-325.44	172,857	0.0033	0.1864	525	14.62	3E+05	0.055	0.2343	-	0.433
GB	-322.37	164,938	0.0032	0.1804	519	28.54	3E+05	1	0.0281	0.107	-

### France 1979

<i>sample</i>	
<i>mean</i>	<i>obs</i>
68,143	9162

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-466.71	84,486	0.0044	0.2422	826	52247	1.617	-	-	-	0.411
lognormal	-414.26	89,542	0.0039	0.2106	714	11.03	0.863	-	-	-	0.458
Weibull	-590.89	84,546	0.0058	0.2945	1082	1.271	91110	-	-	-	0.420
Dagum	-185.54	89,230	0.0013	0.1212	219	2.376	79915	0.698	-	-	0.454
Beta 1	-466.75	84,489	0.0044	0.2422	826	2E+09	1.617	34740	-	-	0.411
Beta 2	-297.01	87,224	0.0027	0.1855	436	1E+05	2.378	4.658	-	-	0.440
GG	-350.71	85,979	0.0034	0.2110	540	0.405	350.5	8.596	-	-	0.431
Singh-Maddala	-215.94	90,260	0.0016	0.1337	285	1.903	76216	1.296	-	-	0.457
GB1	-350.78	85,956	0.0034	0.2111	540	0.407	7E+11	8.544	6004.7	-	0.431
GB2	-133.50	98,557	0.0007	0.0906	117	5.238	69820	0.278	0.3362	-	0.504
GB	-133.50	98,557	0.0007	0.0906	117	5.238	69820	1	0.2779	0.336	-

### France 1984

<i>sample</i>	
<i>mean</i>	<i>obs</i>
113,568	7875

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-193.93	111,602	0.0017	0.1302	240	36459	3.061	-	-	-	0.310
lognormal	-220.37	114,141	0.0022	0.1378	286	11.46	0.604	-	-	-	0.331
Weibull	-328.44	111,146	0.0032	0.2076	532	1.846	1E+05	-	-	-	0.313
Dagum	-179.07	115,599	0.0014	0.1344	211	3.223	1E+05	0.728	-	-	0.336
Beta 1	-193.97	111,687	0.0017	0.1301	239	1E+12	3.048	4E+07	-	-	0.310
Beta 2	-172.11	112,447	0.0015	0.1148	188	4E+05	3.851	13.61	-	-	0.317
GG	-175.77	112,189	0.0016	0.1173	194	0.597	3178	8.077	-	-	0.315
Singh-Maddala	-169.20	113,823	0.0013	0.1182	189	2.494	1E+05	1.619	-	-	0.326
GB1	-175.77	112,189	0.0016	0.1173	194	0.597	1E+13	8.077	471592	-	0.315
GB2	-166.80	113,241	0.0013	0.1112	181	1.937	1E+05	1.444	2.5972	-	0.322
GB	-165.27	111,643	0.0013	0.1142	179	2.485	1E+05	0.961	1.0688	1.202	-

### France 1989

<i>sample</i>	
<i>mean</i>	<i>obs</i>
137,949	5942

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-141.78	136,015	0.0012	0.1203	144	47585	2.858	-	-	-	0.319
lognormal	-159.06	139,450	0.0016	0.1225	177	11.65	0.627	-	-	-	0.343
Weibull	-241.64	135,493	0.0029	0.1941	355	1.777	2E+05	-	-	-	0.323
Dagum	-124.17	141,087	0.0009	0.1095	109	3.149	1E+05	0.706	-	-	0.347
Beta 1	-141.82	136,135	0.0012	0.1204	144	1E+12	2.845	2E+07	-	-	0.320
Beta 2	-122.03	137,212	0.0010	0.0985	100	4E+05	3.635	12.14	-	-	0.328
GG	-125.36	136,845	0.0011	0.1008	106	0.575	3360	8.076	-	-	0.326
Singh-Maddala	-117.92	138,940	0.0008	0.0974	95	2.397	2E+05	1.641	-	-	0.337
GB1	-125.36	136,840	0.0011	0.1008	106	0.575	3E+10	8.084	9972.8	-	0.326
GB2	-117.05	138,402	0.0008	0.0956	92	2.013	2E+05	1.287	2.2561	-	0.334
GB	-117.05	138,402	0.0008	0.0957	92	2.013	2E+05	1	1.2873	2.256	-

## France 1994

<i>sample</i>	
<i>mean</i>	<i>obs</i>
159,343	7880

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-225.73	156,665	0.0022	0.1424	303	81474	1.923	-	-	-	0.382
lognormal	-645.17	169,882	0.0074	0.2723	1364	11.7	0.825	-	-	-	0.441
Weibull	-192.58	155,361	0.0017	0.1341	231	1.498	2E+05	-	-	-	0.370
Dagum	-142.42	158,960	0.0010	0.0961	136	3.554	2E+05	0.377	-	-	0.384
Beta 1	-210.22	155,477	0.0020	0.1432	265	1E+06	1.733	12.26	-	-	0.373
Beta 2	-225.73	156,664	0.0022	0.1424	303	1E+11	1.923	1E+06	-	-	0.382
GG	-192.40	155,416	0.0017	0.1329	230	1.461	2E+05	1.042	-	-	0.371
Singh-Maddala	-183.64	156,183	0.0016	0.1205	217	1.592	6E+05	8.446	-	-	0.374
GB1	-192.40	155,416	0.0017	0.1329	230	1.461	6E+08	1.041	163562	-	0.371
GB2	-141.80	159,799	0.0009	0.0963	135	3.905	2E+05	0.34	0.847	-	0.387
GB	-141.79	157,903	0.0009	0.0965	135	3.954	2E+05	0.997	0.3354	0.816	-

## Germany 1973

<i>sample</i>	
<i>mean</i>	<i>obs</i>
28,312	37614

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-7208.28	29,189	0.0201	0.4622	24536	11717	2.491	-	-	-	0.340
lognormal	-13126.27	32,843	0.0385	0.6442	74310	10.09	0.789	-	-	-	0.423
Weibull	-4951.58	28,525	0.0119	0.3320	17243	1.96	32173	-	-	-	0.298
Dagum	-3502.02	28,100	0.0101	0.3025	8082	6.192	42782	0.241	-	-	0.300
Beta 1	-4910.65	28,227	0.0147	0.4098	11215	68614	1.668	2.386	-	-	0.305
Beta 2	-7208.33	29,188	0.0201	0.4622	24537	3E+09	2.491	3E+05	-	-	0.340
GG	-4029.04	28,098	0.0119	0.3644	8859	3.53	46096	0.416	-	-	0.299
Singh-Maddala	-4951.59	28,526	0.0119	0.3320	17244	1.96	2E+07	2E+05	-	-	0.298
GB1	-4029.05	28,099	0.0119	0.3643	8859	3.53	1E+06	0.416	141201	-	0.299
GB2	-3303.37	28,549	0.0095	0.2712	7595	13.84	36150	0.106	0.2848	-	0.311
GB	-3291.03	27,729	0.0094	0.2668	7580	15.99	35384	1	0.0921	0.229	-

## Germany 1978

<i>sample</i>	
<i>mean</i>	<i>obs</i>
40,277	35731

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-6533.30	41,426	0.0198	0.4547	21103	17460	2.373	-	-	-	0.348
lognormal	-12126.00	46,928	0.0382	0.6387	64703	10.43	0.811	-	-	-	0.434
Weibull	-4555.74	40,492	0.0120	0.3298	15103	1.894	45627	-	-	-	0.306
Dagum	-3213.12	39,907	0.0100	0.2993	7222	5.975	61260	0.241	-	-	0.309
Beta 1	-4516.83	40,069	0.0146	0.4029	10102	1E+05	1.618	2.447	-	-	0.313
Beta 2	-6543.12	41,153	0.0192	0.4421	23016	4E+11	2.448	3E+07	-	-	0.343
GG	-3710.17	39,891	0.0118	0.3603	7986	3.39	66044	0.42	-	-	0.307
Singh-Maddala	-4555.75	40,492	0.0120	0.3298	15102	1.894	3E+07	2E+05	-	-	0.306
GB1	-3710.17	39,891	0.0118	0.3603	7986	3.39	2E+06	0.42	104478	-	0.307
GB2	-3030.90	40,581	0.0094	0.2719	6782	13.11	51602	0.108	0.292	-	0.319
GB	-3022.61	39,382	0.0093	0.2686	6774	14.81	50580	1	0.0961	0.242	-

### Germany 1981

<i>sample</i>	
<i>mean</i>	<i>obs</i>
47,117	2024

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-105.82	47,806	0.0028	0.1507	88	11408	4.191	-	-	-	0.268
lognormal	-145.80	48,710	0.0045	0.1935	190	10.66	0.517	-	-	-	0.285
Weibull	-120.42	47,477	0.0033	0.2036	120	2.233	53605	-	-	-	0.267
Dagum	-93.07	48,373	0.0019	0.1336	63	4.344	54337	0.564	-	-	0.276
Beta 1	-105.26	47,731	0.0028	0.1553	86	7E+05	3.923	51.25	-	-	0.266
Beta 2	-105.82	47,805	0.0028	0.1507	88	3E+10	4.192	3E+06	-	-	0.267
GG	-103.98	47,696	0.0027	0.1586	83	1.287	23069	2.651	-	-	0.265
Singh-Maddala	-95.32	48,028	0.0021	0.1309	67	2.789	63988	2.267	-	-	0.269
GB1	-103.98	47,696	0.0027	0.1586	83	1.287	5E+08	2.651	390171	-	0.265
GB2	-93.00	48,285	0.0019	0.1313	63	4.094	55333	0.605	1.1111	-	0.274
GB	-92.40	47,503	0.0019	0.1280	62	4.785	50212	0.983	0.5132	0.735	-

### Germany 1984

<i>sample</i>	
<i>mean</i>	<i>obs</i>
49,966	4086

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-170.13	50,071	0.0028	0.1615	208	14471	3.46	-	-	-	0.293
lognormal	-299.30	51,516	0.0055	0.2335	554	10.68	0.58	-	-	-	0.318
Weibull	-170.15	49,687	0.0027	0.1743	206	2.045	56084	-	-	-	0.288
Dagum	-125.63	50,527	0.0015	0.1321	118	4.281	60510	0.482	-	-	0.297
Beta 1	-164.12	49,870	0.0027	0.1637	191	4E+05	3.049	19.52	-	-	0.288
Beta 2	-170.18	50,023	0.0028	0.1611	210	1E+11	3.483	8E+06	-	-	0.292
GG	-156.85	49,835	0.0025	0.1598	177	1.496	37041	1.71	-	-	0.287
Singh-Maddala	-139.74	50,148	0.0019	0.1411	146	2.411	81525	3.108	-	-	0.290
GB1	-156.85	49,836	0.0025	0.1598	177	1.495	9E+07	1.711	106852	-	0.287
GB2	-125.34	50,670	0.0015	0.1340	117	4.644	58978	0.439	0.8681	-	0.299
GB	-124.78	49,740	0.0015	0.1333	116	5.122	55596	0.989	0.3971	0.675	-

### Germany 1989

<i>sample</i>	
<i>mean</i>	<i>obs</i>
59,886	2870

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-145.11	60,787	0.0031	0.1599	171	17758	3.423	-	-	-	0.294
lognormal	-260.34	62,750	0.0065	0.2470	499	10.87	0.588	-	-	-	0.322
Weibull	-125.56	60,267	0.0025	0.1660	120	2.063	68035	-	-	-	0.285
Dagum	-105.87	61,143	0.0017	0.1264	83	4.557	76666	0.429	-	-	0.294
Beta 1	-131.71	60,359	0.0028	0.1614	134	3E+05	2.769	10.01	-	-	0.286
Beta 2	-145.11	60,803	0.0031	0.1602	171	6E+11	3.419	4E+07	-	-	0.294
GG	-123.33	60,354	0.0024	0.1560	116	1.781	57718	1.278	-	-	0.285
Singh-Maddala	-116.54	60,606	0.0021	0.1333	105	2.293	1E+05	4.845	-	-	0.287
GB1	-123.33	60,355	0.0024	0.1560	116	1.781	4E+07	1.278	123837	-	0.285
GB2	-105.86	61,188	0.0017	0.1272	83	4.665	76036	0.418	0.9585	-	0.295
GB	-104.78	60,041	0.0016	0.1247	81	5.535	68766	0.984	0.3504	0.621	-

## Germany 1994

<i>sample</i>	
<i>mean</i>	<i>obs</i>
68,016	4613

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-206.95	67,066	0.0030	0.1966	297	27828	2.41	-	-	-	0.345
lognormal	-493.35	71,476	0.0089	0.3333	1062	10.91	0.732	-	-	-	0.395
Weibull	-146.76	66,354	0.0018	0.1506	158	1.735	74466	-	-	-	0.329
Dagum	-104.35	67,045	0.0008	0.0963	73	4.413	94075	0.337	-	-	0.336
Beta 1	-167.61	66,241	0.0023	0.1741	199	3E+05	1.938	6.306	-	-	0.331
Beta 2	-207.08	66,919	0.0030	0.1935	301	3E+11	2.435	1E+07	-	-	0.344
GG	-144.21	66,259	0.0018	0.1548	151	1.946	83904	0.833	-	-	0.329
Singh-Maddala	-146.11	66,443	0.0017	0.1455	159	1.771	5E+05	25.12	-	-	0.330
GB1	-144.22	66,259	0.0018	0.1548	151	1.946	2E+07	0.833	47403	-	0.329
GB2	-99.84	67,886	0.0007	0.0926	64	6.292	84663	0.23	0.5604	-	0.343
GB	-99.40	66,065	0.0007	0.0913	63	6.92	81713	0.996	0.2083	0.465	-

## Hungary 1991

<i>sample</i>	
<i>mean</i>	<i>obs</i>
264,210	1474

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-101.15	257,614	0.0028	0.1863	97	2E+05	1.489	-	-	-	0.426
lognormal	-230.02	300,126	0.0106	0.3660	481	12.12	0.993	-	-	-	0.518
Weibull	-89.16	254,929	0.0021	0.1577	70	1.317	3E+05	-	-	-	0.409
Dagum	-72.32	258,117	0.0011	0.1067	31	3.671	4E+05	0.291	-	-	0.415
Beta 1	-91.63	253,375	0.0025	0.1755	71	1E+06	1.286	5.835	-	-	0.408
Beta 2	-101.15	257,614	0.0028	0.1863	97	1E+12	1.489	7E+06	-	-	0.426
GG	-84.19	253,470	0.0020	0.1587	54	1.735	4E+05	0.652	-	-	0.405
Singh-Maddala	-89.16	254,929	0.0021	0.1577	70	1.317	3E+09	2E+05	-	-	0.409
GB1	-84.19	253,472	0.0020	0.1587	54	1.735	4E+08	0.652	189315	-	0.405
GB2	-70.13	265,300	0.0010	0.0896	26	6.225	3E+05	0.166	0.4398	-	0.428
GB	-69.86	250,674	0.0010	0.0862	26	7.421	3E+05	0.998	0.1385	0.334	-

## Ireland 1987

<i>sample</i>	
<i>mean</i>	<i>obs</i>
13,966	2482

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-116.40	14,412	0.0022	0.1644	113	9450	1.525	-	-	-	0.421
lognormal	-309.38	16,536	0.0103	0.3597	591	9.243	0.97	-	-	-	0.507
Weibull	-96.50	14,269	0.0014	0.1241	73	1.334	15528	-	-	-	0.405
Dagum	-84.06	14,558	0.0009	0.1134	45	3.713	22790	0.287	-	-	0.413
Beta 1	-94.83	14,151	0.0013	0.1356	67	65252	1.268	4.578	-	-	0.400
Beta 2	-116.41	14,392	0.0022	0.1637	114	8E+10	1.531	9E+06	-	-	0.421
GG	-87.64	14,194	0.0010	0.1184	53	1.816	22033	0.624	-	-	0.400
Singh-Maddala	-96.50	14,270	0.0014	0.1241	73	1.334	3E+08	5E+05	-	-	0.405
GB1	-87.64	14,194	0.0010	0.1184	53	1.816	1E+07	0.624	102250	-	0.400
GB2	-83.34	14,401	0.0009	0.1082	44	3.058	26538	0.354	1.548	-	0.408
GB	-80.57	13,973	0.0008	0.1008	38	4.425	17936	0.969	0.2438	0.526	-

### Israel 1979

<i>sample</i>	
<i>mean</i>	<i>obs</i>
31,013	1941

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-78.36	32,455	0.0010	0.1068	38	14447	2.246	-	-	-	0.356
lognormal	-159.86	34,381	0.0049	0.2465	229	10.17	0.746	-	-	-	0.402
Weibull	-69.97	32,201	0.0006	0.0846	21	1.636	35986	-	-	-	0.345
Dagum	-70.62	32,993	0.0006	0.0913	22	3.873	44743	0.373	-	-	0.359
Beta 1	-71.78	32,173	0.0007	0.0914	24	2E+05	1.916	8.951	-	-	0.346
Beta 2	-78.37	32,490	0.0010	0.1075	38	1E+11	2.237	7E+06	-	-	0.357
GG	-69.79	32,220	0.0006	0.0837	20	1.549	33526	1.094	-	-	0.346
Singh-Maddala	-69.26	32,283	0.0005	0.0790	19	1.696	2E+05	13.98	-	-	0.347
GB1	-69.79	32,220	0.0006	0.0837	20	1.549	5E+07	1.094	93294	-	0.346
GB2	-68.22	32,448	0.0005	0.0721	17	2.471	63829	0.628	2.7932	-	0.350
GB	-68.06	32,147	0.0005	0.0711	17	2.894	45039	0.919	0.5298	1.242	-

### Israel 1986

<i>sample</i>	
<i>mean</i>	<i>obs</i>
22,902	3942

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-76.01	23,179	0.0002	0.0590	20	11961	1.938	-	-	-	0.380
lognormal	-139.84	24,486	0.0018	0.1507	154	9.792	0.793	-	-	-	0.425
Weibull	-101.32	23,087	0.0009	0.1121	71	1.46	25486	-	-	-	0.378
Dagum	-83.00	24,291	0.0004	0.0757	34	2.884	27121	0.53	-	-	0.409
Beta 1	-76.01	23,170	0.0002	0.0589	20	1E+11	1.94	1E+07	-	-	0.380
Beta 2	-73.24	23,309	0.0002	0.0472	14	2E+05	2.105	20.11	-	-	0.386
GG	-73.72	23,260	0.0002	0.0488	15	0.828	6713	2.701	-	-	0.385
Singh-Maddala	-73.62	23,518	0.0002	0.0474	15	1.725	41376	2.951	-	-	0.391
GB1	-73.72	23,260	0.0002	0.0488	15	0.828	8E+11	2.702	5E+06	-	0.385
GB2	-72.86	23,389	0.0002	0.0450	13	1.314	66987	1.449	6.2257	-	0.388
GB	-72.83	23,290	0.0002	0.0455	13	1.485	37307	0.896	1.2527	2.772	-

### Israel 1992

<i>sample</i>	
<i>mean</i>	<i>obs</i>
66,271	4057

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-82.34	66,672	0.0004	0.0683	32	36345	1.834	-	-	-	0.389
lognormal	-145.48	70,765	0.0019	0.1523	169	10.83	0.818	-	-	-	0.437
Weibull	-110.46	66,437	0.0011	0.1250	89	1.411	72979	-	-	-	0.388
Dagum	-77.30	70,110	0.0003	0.0604	22	2.761	76235	0.547	-	-	0.421
Beta 1	-82.34	66,670	0.0004	0.0683	32	4E+11	1.835	1E+07	-	-	0.389
Beta 2	-76.11	67,254	0.0003	0.0541	19	4E+05	2.063	13.67	-	-	0.398
GG	-77.91	67,001	0.0003	0.0587	22	0.778	16374	2.857	-	-	0.396
Singh-Maddala	-73.06	68,012	0.0002	0.0446	13	1.708	1E+05	2.593	-	-	0.405
GB1	-77.91	67,002	0.0003	0.0587	22	0.778	4E+12	2.857	3E+06	-	0.396
GB2	-73.02	68,115	0.0002	0.0451	13	1.79	1E+05	0.939	2.3396	-	0.405
GB	-72.55	66,680	0.0002	0.0449	12	2.223	72600	0.955	0.731	1.139	-

### Israel 1997

<i>sample</i>	
<i>mean</i>	<i>obs</i>
116,473	4072

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-82.40	114,834	0.0004	0.0729	31	73068	1.572	-	-	-	0.416
lognormal	-208.37	125,636	0.0033	0.2123	313	11.33	0.91	-	-	-	0.480
Weibull	-91.86	114,320	0.0006	0.0931	51	1.306	1E+05	-	-	-	0.412
Dagum	-75.98	119,947	0.0002	0.0572	19	2.829	1E+05	0.443	-	-	0.440
Beta 1	-82.40	114,819	0.0004	0.0729	31	3E+11	1.572	5E+06	-	-	0.416
Beta 2	-81.75	115,179	0.0004	0.0694	30	3E+06	1.622	38.88	-	-	0.419
GG	-82.36	114,890	0.0004	0.0723	31	0.98	69585	1.625	-	-	0.417
Singh-Maddala	-77.09	116,022	0.0003	0.0579	21	1.464	3E+05	4.389	-	-	0.422
GB1	-82.36	114,892	0.0004	0.0723	31	0.98	6E+11	1.625	6E+06	-	0.417
GB2	-74.37	117,662	0.0002	0.0527	16	2.185	2E+05	0.602	1.6439	-	0.430
GB	-73.97	114,444	0.0002	0.0508	15	2.616	1E+05	0.965	0.4931	0.94	-

### Italy 1986

<i>sample</i>	
<i>mean</i>	<i>obs</i>
21,086	6016

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-244.65	21,119	0.0031	0.1987	349	6292	3.357	-	-	-	0.297
lognormal	-305.49	21,585	0.0039	0.2251	492	9.813	0.578	-	-	-	0.317
Weibull	-325.35	21,011	0.0041	0.2556	535	1.958	23698	-	-	-	0.298
Dagum	-240.27	21,704	0.0031	0.2004	335	3.507	22273	0.673	-	-	0.317
Beta 1	-244.65	21,119	0.0031	0.1987	349	1E+10	3.357	2E+06	-	-	0.297
Beta 2	-241.02	21,184	0.0030	0.1928	341	2E+05	3.723	30.55	-	-	0.300
GG	-242.69	21,155	0.0031	0.1954	345	0.842	3359	4.617	-	-	0.299
Singh-Maddala	-233.17	21,391	0.0029	0.1866	322	2.546	25821	1.861	-	-	0.307
GB1	-243.59	21,128	0.0031	0.1972	347	0.948	2E+11	3.704	1E+07	-	0.299
GB2	-232.91	21,357	0.0029	0.1854	322	2.334	26989	1.128	2.2071	-	0.306
GB	-227.51	20,945	0.0028	0.1857	313	3.393	19554	0.97	0.7382	0.797	-

### Italy 1995

<i>sample</i>	
<i>mean</i>	<i>obs</i>
33,909	5479

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-239.59	33,268	0.0030	0.1905	364	12533	2.654	-	-	-	0.330
lognormal	-398.56	34,613	0.0062	0.2636	713	10.23	0.67	-	-	-	0.365
Weibull	-254.03	33,051	0.0029	0.1971	418	1.762	37124	-	-	-	0.325
Dagum	-215.65	33,960	0.0025	0.1785	317	3.598	40039	0.499	-	-	0.342
Beta 1	-237.85	33,187	0.0029	0.1926	364	5E+05	2.51	36.35	-	-	0.328
Beta 2	-239.59	33,265	0.0030	0.1905	364	1E+11	2.656	9E+06	-	-	0.330
GG	-234.41	33,165	0.0028	0.1907	359	1.244	21372	1.817	-	-	0.326
Singh-Maddala	-220.76	33,413	0.0026	0.1753	327	2.053	59523	3.283	-	-	0.331
GB1	-234.41	33,166	0.0028	0.1907	359	1.244	4E+08	1.817	192516	-	0.326
GB2	-214.55	33,717	0.0025	0.1732	315	3.11	43060	0.591	1.3232	-	0.338
GB	-209.14	32,791	0.0024	0.1693	300	4.173	34151	0.979	0.4351	0.618	-

### Mexico 1984

sample	
mean	obs
507	4440

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-122.13	484	0.0011	0.1244	117	325.4	1.488	-	-	-	0.426
lognormal	-217.06	530	0.0032	0.1965	329	5.839	0.932	-	-	-	0.490
Weibull	-143.47	483	0.0016	0.1491	163	1.25	518.6	-	-	-	0.426
Dagum	-90.73	513	0.0005	0.0770	50	2.536	562.6	0.516	-	-	0.462
Beta 1	-122.14	484	0.0011	0.1244	117	1E+07	1.488	34029	-	-	0.426
Beta 2	-109.58	491	0.0009	0.1081	89	2461	1.703	9.531	-	-	0.439
GG	-115.46	487	0.0010	0.1178	102	0.773	153	2.314	-	-	0.434
Singh-Maddala	-99.55	498	0.0007	0.0918	68	1.516	818.4	2.633	-	-	0.446
GB1	-115.46	487	0.0010	0.1178	102	0.773	8E+07	2.315	26247	-	0.434
GB2	-90.55	517	0.0005	0.0780	49	2.73	543.5	0.474	0.8834	-	0.466
GB	-88.57	480	0.0004	0.0735	45	3.428	467.1	0.991	0.371	0.553	-

### Mexico 1989

sample	
mean	obs
10,032	10812

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-271.44	9,224	0.0017	0.1572	414	6741	1.368	-	-	-	0.441
lognormal	-189.15	9,955	0.0010	0.1201	234	8.751	0.953	-	-	-	0.500
Weibull	-352.67	9,230	0.0026	0.1898	569	1.167	9740	-	-	-	0.448
Dagum	-108.88	10,317	0.0003	0.0601	66	2.085	8492	0.702	-	-	0.512
Beta 1	-271.47	9,234	0.0017	0.1579	413	9E+10	1.364	1E+07	-	-	0.442
Beta 2	-112.51	9,644	0.0003	0.0720	73	14841	1.955	4.009	-	-	0.479
GG	-129.83	9,462	0.0005	0.0846	107	0.368	21.92	8.515	-	-	0.468
Singh-Maddala	-104.60	9,932	0.0003	0.0586	58	1.595	9633	1.586	-	-	0.494
GB1	-129.85	9,461	0.0005	0.0846	108	0.368	2E+11	8.528	4955.3	-	0.468
GB2	-104.59	9,944	0.0003	0.0583	58	1.616	9573	0.981	1.5493	-	0.494
GB	-104.59	9,944	0.0003	0.0583	58	1.616	9573	1	0.9809	1.549	-

### Mexico 1992

sample	
mean	obs
20,135	9881

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-322.84	15,824	0.0024	0.1778	532	12578	1.258	-	-	-	0.456
lognormal	-245.56	17,303	0.0017	0.1459	358	9.255	1.003	-	-	-	0.522
Weibull	-379.76	15,847	0.0030	0.2024	638	1.108	16465	-	-	-	0.465
Dagum	-107.74	17,715	0.0003	0.0636	67	2.027	14529	0.682	-	-	0.529
Beta 1	-322.84	15,828	0.0024	0.1780	531	2E+11	1.257	1E+07	-	-	0.457
Beta 2	-142.88	16,664	0.0007	0.0974	138	24287	1.811	3.639	-	-	0.499
GG	-176.47	16,257	0.0010	0.1212	205	0.38	67.05	7.279	-	-	0.485
Singh-Maddala	-118.76	17,275	0.0004	0.0744	89	1.543	15640	1.524	-	-	0.516
GB1	-176.49	16,256	0.0010	0.1212	205	0.38	3E+12	7.281	11210	-	0.485
GB2	-105.26	18,395	0.0003	0.0619	62	2.438	13645	0.543	0.75	-	0.546
GB	-103.20	16,005	0.0003	0.0581	57	2.831	12518	0.995	0.4602	0.566	-



### Mexico 1994

sample	
mean	obs
24,260	11931

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-357.10	18,518	0.0022	0.1750	597	16341	1.133	-	-	-	0.476
lognormal	-358.83	20,856	0.0022	0.1620	605	9.362	1.08	-	-	-	0.555
Weibull	-388.21	18,548	0.0025	0.1941	651	1.047	18889	-	-	-	0.484
Dagum	-115.67	20,795	0.0003	0.0645	79	1.998	18146	0.61	-	-	0.548
Beta 1	-357.10	18,516	0.0022	0.1749	597	1E+11	1.134	9E+06	-	-	0.476
Beta 2	-168.07	19,519	0.0008	0.1066	185	34036	1.536	3.679	-	-	0.518
GG	-213.59	18,989	0.0011	0.1324	277	0.467	634.8	4.342	-	-	0.503
Singh-Maddala	-140.73	20,110	0.0005	0.0859	131	1.393	21134	1.782	-	-	0.532
GB1	-213.60	18,988	0.0011	0.1324	277	0.467	1E+12	4.341	22890	-	0.498
GB2	-108.30	22,262	0.0003	0.0594	64	2.68	16251	0.43	0.6368	-	0.577
GB	-104.67	18,563	0.0002	0.0534	56	3.15	14939	0.997	0.3614	0.479	-

### Mexico 1996

sample	
mean	obs
31,341	13100

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-446.44	25,710	0.0026	0.1918	791	20260	1.269	-	-	-	0.455
lognormal	-287.09	27,943	0.0015	0.1443	440	9.744	0.994	-	-	-	0.518
Weibull	-529.21	25,754	0.0034	0.2211	943	1.111	26780	-	-	-	0.464
Dagum	-137.16	28,929	0.0004	0.0735	122	2	22760	0.715	-	-	0.530
Beta 1	-446.54	25,659	0.0026	0.1905	794	3E+11	1.275	1E+07	-	-	0.454
Beta 2	-178.30	27,189	0.0007	0.1087	207	35170	1.893	3.449	-	-	0.500
GG	-220.51	26,491	0.0011	0.1331	293	0.328	18.69	9.817	-	-	0.486
Singh-Maddala	-148.23	28,296	0.0005	0.0851	145	1.579	23878	1.428	-	-	0.519
GB1	-220.57	26,488	0.0011	0.1331	294	0.327	3E+12	9.839	4850.3	-	0.486
GB2	-133.91	30,103	0.0004	0.0712	115	2.436	21471	0.559	0.7394	-	0.548
GB	-128.21	25,713	0.0004	0.0639	103	3.049	19421	0.996	0.4331	0.501	-

### Netherlands 1983

sample	
mean	obs
54,318	3090

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-155.70	51,251	0.0036	0.1814	177	11729	4.37	-	-	-	0.262
lognormal	-196.33	52,078	0.0049	0.1835	284	10.73	0.502	-	-	-	0.277
Weibull	-201.33	50,922	0.0047	0.2487	284	2.247	57492	-	-	-	0.265
Dagum	-129.22	51,957	0.0025	0.1542	126	4.109	54395	0.671	-	-	0.273
Beta 1	-155.71	51,267	0.0036	0.1814	177	6E+10	4.358	5E+06	-	-	0.263
Beta 2	-155.17	51,312	0.0036	0.1745	176	8E+05	4.624	72.52	-	-	0.263
GG	-155.69	51,245	0.0036	0.1821	177	1.013	12276	4.262	-	-	0.262
Singh-Maddala	-133.86	51,784	0.0026	0.1484	136	3.065	57678	1.615	-	-	0.269
GB1	-155.70	51,251	0.0036	0.1814	177	1	6E+10	4.371	5E+06	-	0.262
GB2	-128.59	52,222	0.0024	0.1603	126	4.729	52568	0.568	0.7903	-	0.277
GB	-123.89	50,682	0.0022	0.1537	116	6.097	48013	0.994	0.437	0.476	-

### Netherlands 1987

sample	
mean	obs
57,209	2792

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-134.19	54,959	0.0029	0.1864	138	13448	4.087	-	-	-	0.271
lognormal	-159.01	55,863	0.0038	0.1836	199	10.8	0.518	-	-	-	0.286
Weibull	-183.71	54,643	0.0043	0.2542	253	2.156	61701	-	-	-	0.275
Dagum	-113.59	55,955	0.0020	0.1592	98	3.848	56776	0.713	-	-	0.285
Beta 1	-134.19	54,970	0.0029	0.1862	138	5E+11	4.08	3E+07	-	-	0.271
Beta 2	-131.58	55,127	0.0029	0.1756	132	3E+05	4.683	29.53	-	-	0.274
GG	-133.24	55,041	0.0029	0.1800	136	0.832	6555	5.775	-	-	0.272
Singh-Maddala	-115.89	55,724	0.0021	0.1561	103	3.006	59474	1.493	-	-	0.281
GB1	-133.24	55,040	0.0029	0.1801	136	0.833	6E+09	5.761	96292	-	0.272
GB2	-113.43	56,124	0.0020	0.1610	98	4.156	55715	0.65	0.8789	-	0.288
GB	-109.20	54,392	0.0018	0.1558	90	5.475	50197	0.993	0.4859	0.504	-

### Netherlands 1991

sample	
mean	obs
62,634	2971

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-500.76	62,002	0.0163	0.4123	1211	28260	2.194	-	-	-	0.360
lognormal	-936.73	70,584	0.0343	0.6074	3512	10.81	0.839	-	-	-	0.447
Weibull	-356.09	60,668	0.0093	0.2800	881	1.801	68223	-	-	-	0.319
Dagum	-248.86	59,947	0.0069	0.2456	401	6.175	96338	0.214	-	-	0.321
Beta 1	-323.12	59,885	0.0099	0.3315	563	1E+05	1.472	2.128	-	-	0.321
Beta 2	-500.84	61,865	0.0162	0.4090	1235	5E+11	2.215	2E+07	-	-	0.358
GG	-271.33	59,722	0.0078	0.2870	443	3.618	1E+05	0.364	-	-	0.317
Singh-Maddala	-356.10	60,668	0.0093	0.2800	881	1.801	8E+07	4E+05	-	-	0.319
GB1	-271.33	59,722	0.0078	0.2870	443	3.618	3E+06	0.364	126844	-	0.317
GB2	-245.09	60,491	0.0068	0.2460	392	9.022	85755	0.145	0.5037	-	0.326
GB	-243.69	59,106	0.0067	0.2410	390	10.88	80757	0.995	0.1208	0.346	-

### Netherlands 1994

sample	
mean	obs
67,120	3565

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-522.19	69,605	0.0137	0.4083	1163	35221	1.976	-	-	-	0.377
lognormal	-1043.73	80,697	0.0313	0.6138	3407	10.9	0.891	-	-	-	0.471
Weibull	-369.98	68,112	0.0077	0.2793	872	1.689	76304	-	-	-	0.337
Dagum	-230.55	67,231	0.0049	0.2296	350	6.263	1E+05	0.194	-	-	0.335
Beta 1	-304.78	67,068	0.0072	0.3102	506	2E+05	1.337	1.961	-	-	0.335
Beta 2	-522.19	69,605	0.0137	0.4083	1163	6E+10	1.976	2E+06	-	-	0.377
GG	-250.76	66,928	0.0055	0.2684	388	3.733	1E+05	0.325	-	-	0.331
Singh-Maddala	-369.98	68,113	0.0077	0.2793	872	1.689	2E+08	5E+05	-	-	0.337
GB1	-250.76	66,928	0.0055	0.2684	388	3.733	3E+06	0.325	115873	-	0.155
GB2	-227.29	67,806	0.0048	0.2247	343	9.014	1E+05	0.134	0.5133	-	0.340
GB	-223.88	66,106	0.0047	0.2124	336	12.88	92167	0.997	0.0936	0.275	-

## Norway 1979

<i>sample</i>	
<i>mean</i>	<i>obs</i>
94,793	8272

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-944.48	130,072	0.0121	0.4228	1762	1E+05	1.201	-	-	-	0.465
lognormal	-1610.98	160,461	0.0190	0.5135	4166	11.35	1.131	-	-	-	0.576
Weibull	-945.79	129,581	0.0121	0.4238	1762	1.118	1E+05	-	-	-	0.462
Dagum	-625.37	128,842	0.0072	0.3232	1133	2.761	2E+05	0.39	-	-	0.467
Beta 1	-944.48	130,078	0.0121	0.4228	1762	4E+11	1.201	4E+06	-	-	0.465
Beta 2	-925.05	131,661	0.0118	0.4142	1740	1E+06	1.296	12.12	-	-	0.476
GG	-943.57	129,839	0.0121	0.4226	1760	1.046	1E+05	1.118	-	-	0.463
Singh-Maddala	-859.19	132,684	0.0103	0.3815	1691	1.327	3E+05	3.391	-	-	0.474
GB1	-943.57	129,837	0.0121	0.4225	1760	1.046	2E+11	1.118	3E+06	-	0.463
GB2	-385.43	147,016	0.0040	0.2162	636	42.08	1E+05	0.024	0.0415	-	0.528
GB	-385.43	147,016	0.0040	0.2162	636	42.08	1E+05	1	0.0238	0.041	-

## Norway 1986

<i>sample</i>	
<i>mean</i>	<i>obs</i>
204,406	4250

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-540.45	235,147	0.0113	0.3403	1233	1E+05	1.97	-	-	-	0.378
lognormal	-1136.00	271,334	0.0273	0.5384	3843	12.12	0.887	-	-	-	0.469
Weibull	-372.86	230,343	0.0060	0.2292	889	1.672	3E+05	-	-	-	0.339
Dagum	-219.58	227,747	0.0038	0.1908	320	6.184	4E+05	0.195	-	-	0.337
Beta 1	-306.20	226,973	0.0061	0.2746	503	6E+05	1.346	2.062	-	-	0.337
Beta 2	-540.54	235,143	0.0113	0.3403	1234	2E+09	1.97	13434	-	-	0.378
GG	-245.28	226,677	0.0045	0.2315	369	3.598	4E+05	0.336	-	-	0.334
Singh-Maddala	-372.86	230,347	0.0060	0.2291	889	1.672	6E+08	4E+05	-	-	0.339
GB1	-245.28	226,682	0.0045	0.2315	369	3.597	1E+07	0.336	87458	-	0.334
GB2	-213.57	230,296	0.0037	0.1925	305	10.25	3E+05	0.116	0.423	-	0.343
GB	-210.84	224,035	0.0036	0.1860	300	14.2	3E+05	0.998	0.0839	0.252	-

## Norway 1991

<i>sample</i>	
<i>mean</i>	<i>obs</i>
262,892	7361

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-609.80	328,538	0.0070	0.2787	1312	2E+05	1.842	-	-	-	0.389
lognormal	-1498.46	377,678	0.0197	0.4721	4602	12.43	0.904	-	-	-	0.477
Weibull	-409.31	322,893	0.0038	0.1883	878	1.564	4E+05	-	-	-	0.358
Dagum	-231.58	321,775	0.0023	0.1508	325	5.295	5E+05	0.219	-	-	0.357
Beta 1	-352.25	318,933	0.0041	0.2161	578	9E+05	1.335	2.539	-	-	0.355
Beta 2	-611.86	325,701	0.0067	0.2707	1388	2E+12	1.898	1E+07	-	-	0.384
GG	-272.92	319,099	0.0029	0.1858	407	2.915	6E+05	0.404	-	-	0.352
Singh-Maddala	-409.31	322,896	0.0038	0.1883	878	1.564	1E+09	4E+05	-	-	0.358
GB1	-272.92	319,101	0.0029	0.1858	407	2.915	3E+07	0.404	127778	-	0.352
GB2	-226.62	324,884	0.0022	0.1467	313	7.291	5E+05	0.157	0.5665	-	0.362
GB	-225.21	316,781	0.0022	0.1448	310	8.56	5E+05	0.992	0.1336	0.406	-

## Norway 1995

<i>sample</i>	
<i>mean</i>	<i>obs</i>
285,297	8027

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-784.90	350,548	0.0083	0.3135	1752	2E+05	1.675	-	-	-	0.405
lognormal	-1892.17	417,906	0.0230	0.5204	5847	12.47	0.974	-	-	-	0.509
Weibull	-554.06	343,799	0.0047	0.2267	1321	1.5	4E+05	-	-	-	0.370
Dagum	-258.06	339,505	0.0023	0.1597	390	5.817	6E+05	0.185	-	-	0.363
Beta 1	-408.84	337,845	0.0042	0.2314	709	9E+05	1.199	2.094	-	-	0.363
Beta 2	-785.14	351,650	0.0084	0.3160	1728	4E+12	1.659	2E+07	-	-	0.407
GG	-307.56	337,758	0.0029	0.1939	489	3.362	6E+05	0.321	-	-	0.360
Singh-Maddala	-554.06	343,802	0.0047	0.2267	1321	1.5	2E+09	5E+05	-	-	0.370
GB1	-307.56	337,760	0.0029	0.1939	489	3.363	2E+07	0.321	106385	-	0.360
GB2	-237.94	345,601	0.0021	0.1468	344	13.25	5E+05	0.08	0.2811	-	0.373
GB	-231.54	332,590	0.0020	0.1360	331	20.67	5E+05	1	0.0512	0.156	-

## Poland 1986

<i>sample</i>	
<i>mean</i>	<i>obs</i>
477,059	9083

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-1135.62	477,689	0.0134	0.3612	2576	3E+05	1.489	-	-	-	0.426
lognormal	-2668.53	609,728	0.0352	0.5997	8670	12.74	1.079	-	-	-	0.555
Weibull	-875.59	467,276	0.0083	0.2653	2335	1.418	5E+05	-	-	-	0.387
Dagum	-436.41	457,546	0.0049	0.1930	731	5.91	8E+05	0.168	-	-	0.377
Beta 1	-617.32	456,533	0.0073	0.2726	1096	1E+06	1.073	1.831	-	-	0.377
Beta 2	-1136.21	475,256	0.0132	0.3559	2638	6E+12	1.509	2E+07	-	-	0.423
GG	-485.76	455,314	0.0055	0.2288	823	3.574	9E+05	0.278	-	-	0.373
Singh-Maddala	-875.60	467,284	0.0083	0.2653	2335	1.418	7E+09	7E+05	-	-	0.387
GB1	-485.76	455,318	0.0055	0.2288	823	3.574	2E+07	0.278	79526	-	0.373
GB2	-421.29	464,383	0.0047	0.1804	696	10.93	7E+05	0.09	0.3637	-	0.384
GB	-419.19	450,040	0.0046	0.1746	692	14.01	7E+05	0.999	0.0701	0.249	-

## Poland 1992

<i>sample</i>	
<i>mean</i>	<i>obs</i>
43,462	4179

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-299.79	43,445	0.0066	0.2014	525	20755	2.093	-	-	-	0.368
lognormal	-644.79	47,533	0.0141	0.3422	1916	10.44	0.806	-	-	-	0.431
Weibull	-236.14	42,925	0.0051	0.1666	345	1.614	47915	-	-	-	0.349
Dagum	-206.72	43,513	0.0045	0.1599	264	4.17	62888	0.323	-	-	0.359
Beta 1	-251.85	42,759	0.0060	0.1940	358	2E+05	1.656	4.89	-	-	0.349
Beta 2	-299.79	43,445	0.0066	0.2014	525	7E+09	2.093	3E+05	-	-	0.368
GG	-227.23	42,787	0.0053	0.1829	302	2.005	59858	0.711	-	-	0.348
Singh-Maddala	-236.14	42,925	0.0051	0.1666	345	1.614	6E+07	1E+05	-	-	0.349
GB1	-227.23	42,788	0.0053	0.1829	302	2.005	2E+07	0.711	67741	-	0.348
GB2	-206.71	43,489	0.0045	0.1594	264	4.118	63293	0.327	1.025	-	0.358
GB	-204.95	42,438	0.0044	0.1573	261	4.936	54185	0.976	0.2728	0.605	-

**Poland 1995**

sample	
mean	obs
9,651	21091

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-245.44	9,435	0.0007	0.0973	336	3993	2.363	-	-	-	0.348
lognormal	-252.94	9,747	0.0008	0.1050	345	8.942	0.697	-	-	-	0.378
Weibull	-558.07	9,408	0.0024	0.1599	960	1.602	10495	-	-	-	0.351
Dagum	-211.86	9,922	0.0006	0.0923	262	2.845	9592	0.693	-	-	0.384
Beta 1	-242.53	9,456	0.0007	0.0986	328	8E+11	2.34	2E+08	-	-	0.350
Beta 2	-148.55	9,560	0.0003	0.0667	131	24840	3.086	9.019	-	-	0.361
GG	-151.15	9,525	0.0003	0.0683	136	0.494	102.8	8.851	-	-	0.359
Singh-Maddala	-171.18	9,686	0.0004	0.0748	180	2.118	11171	1.753	-	-	0.370
GB1	-151.17	9,525	0.0003	0.0683	136	0.491	1E+09	8.928	3139.4	-	0.359
GB2	-148.54	9,563	0.0003	0.0666	131	1.033	23109	2.937	8.254	-	0.362
GB	-148.54	9,563	0.0003	0.0666	131	1.033	23109	1	2.9374	8.254	-

**Taiwan 1981**

sample	
mean	obs
256,816	14997

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-329.60	252,769	0.0016	0.1558	526	61915	4.083	-	-	-	0.271
lognormal	-227.66	255,851	0.0010	0.1007	303	12.32	0.511	-	-	-	0.282
Weibull	-782.11	251,699	0.0048	0.2645	1467	2.103	3E+05	-	-	-	0.281
Dagum	-164.88	259,616	0.0006	0.0853	174	3.472	2E+05	0.929	-	-	0.293
Beta 1	-329.60	252,770	0.0016	0.1558	526	3E+10	4.082	6E+05	-	-	0.271
Beta 2	-206.66	255,342	0.0008	0.0964	260	3E+05	6.835	9.853	-	-	0.280
GG	-219.94	254,847	0.0009	0.1033	287	0.205	7E-05	91.21	-	-	0.279
Singh-Maddala	-164.96	259,241	0.0006	0.0836	174	3.296	2E+05	1.082	-	-	0.292
GB1	-220.19	254,852	0.0009	0.1034	287	0.159	1E+11	133.9	967.01	-	0.278
GB2	-164.86	259,466	0.0006	0.0847	174	3.416	2E+05	0.949	1.0256	-	0.293
GB	-152.39	251,506	0.0005	0.0785	148	4.719	2E+05	0.994	0.641	0.567	-

**Taiwan 1986**

sample	
mean	obs
337,994	15911

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-343.15	331,699	0.0017	0.1572	534	92971	3.568	-	-	-	0.288
lognormal	-393.08	337,746	0.0020	0.1327	654	12.58	0.555	-	-	-	0.305
Weibull	-689.02	330,141	0.0038	0.2406	1274	1.985	4E+05	-	-	-	0.295
Dagum	-174.10	338,847	0.0006	0.0883	190	3.549	3E+05	0.734	-	-	0.306
Beta 1	-343.27	331,458	0.0017	0.1571	535	2E+12	3.587	2E+07	-	-	0.288
Beta 2	-287.08	333,931	0.0014	0.1285	414	1E+06	4.626	14.54	-	-	0.295
GG	-306.47	333,169	0.0015	0.1376	452	0.595	7090	9.561	-	-	0.293
Singh-Maddala	-191.90	338,368	0.0007	0.0919	227	2.873	3E+05	1.359	-	-	0.304
GB1	-306.47	333,170	0.0015	0.1376	452	0.595	2E+13	9.561	478262	-	0.293
GB2	-164.45	342,857	0.0005	0.0840	171	4.698	3E+05	0.521	0.6513	-	0.314
GB	-149.09	328,600	0.0004	0.0690	140	5.994	3E+05	0.997	0.3988	0.43	-

### Taiwan 1991

sample	
mean	obs
592,997	15756

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-231.49	579,691	0.0010	0.1214	302	2E+05	3.296	-	-	-	0.299
lognormal	-409.77	593,192	0.0020	0.1488	709	13.12	0.584	-	-	-	0.321
Weibull	-451.66	576,678	0.0024	0.1927	762	1.934	7E+05	-	-	-	0.301
Dagum	-140.57	591,983	0.0004	0.0792	123	3.607	6E+05	0.634	-	-	0.315
Beta 1	-231.55	580,037	0.0010	0.1215	302	1E+12	3.283	7E+06	-	-	0.300
Beta 2	-219.01	581,650	0.0009	0.1119	278	4E+06	3.683	28.05	-	-	0.303
GG	-226.68	580,639	0.0010	0.1179	293	0.853	99630	4.418	-	-	0.301
Singh-Maddala	-156.27	587,670	0.0005	0.0837	154	2.562	7E+05	1.784	-	-	0.309
GB1	-227.56	580,153	0.0010	0.1194	294	0.916	1E+13	3.877	2E+07	-	0.301
GB2	-139.96	593,814	0.0004	0.0784	122	3.888	6E+05	0.579	0.8845	-	0.317
GB	-114.96	572,267	0.0002	0.0524	72	5.636	5E+05	0.995	0.3846	0.45	-

### Taiwan 1995

sample	
mean	obs
767,530	13619

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-374.67	770,304	0.0023	0.1452	608	3E+05	2.634	-	-	-	0.332
lognormal	-940.01	806,595	0.0060	0.2510	2116	13.37	0.681	-	-	-	0.370
Weibull	-338.26	764,176	0.0021	0.1684	509	1.774	9E+05	-	-	-	0.323
Dagum	-216.03	778,760	0.0011	0.1086	272	3.864	1E+06	0.444	-	-	0.335
Beta 1	-350.70	766,134	0.0022	0.1572	540	6E+06	2.339	15.38	-	-	0.326
Beta 2	-374.67	770,300	0.0023	0.1452	608	3E+12	2.634	1E+07	-	-	0.332
GG	-322.17	765,668	0.0019	0.1562	479	1.482	7E+05	1.354	-	-	0.324
Singh-Maddala	-281.11	770,456	0.0016	0.1275	409	2.006	2E+06	4.176	-	-	0.327
GB1	-322.17	765,674	0.0019	0.1562	479	1.482	2E+09	1.354	158040	-	0.324
GB2	-211.46	785,772	0.0010	0.1071	264	4.702	9E+05	0.357	0.7148	-	0.341
GB	-183.05	753,869	0.0008	0.0879	206	6.761	8E+05	0.996	0.2459	0.37	-

### Russia 1992

sample	
mean	obs
84,594	4851

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-136.11	74,951	0.0015	0.1118	137	59220	1.266	-	-	-	0.455
lognormal	-460.01	88,787	0.0074	0.2997	1007	10.82	1.075	-	-	-	0.553
Weibull	-127.35	74,528	0.0013	0.1102	117	1.169	78685	-	-	-	0.447
Dagum	-104.93	77,329	0.0009	0.0883	71	2.93	1E+05	0.342	-	-	0.466
Beta 1	-132.53	74,415	0.0015	0.1159	126	1E+06	1.201	15.68	-	-	0.448
Beta 2	-136.36	75,359	0.0015	0.1155	136	1E+12	1.251	2E+07	-	-	0.458
GG	-125.72	74,357	0.0013	0.1163	111	1.285	91217	0.861	-	-	0.445
Singh-Maddala	-127.13	74,656	0.0013	0.1064	118	1.184	2E+06	41.85	-	-	0.448
GB1	-125.72	74,358	0.0013	0.1163	111	1.284	2E+09	0.861	286686	-	0.445
GB2	-104.93	77,348	0.0009	0.0883	71	2.938	1E+05	0.341	0.995	-	0.466
GB	-104.01	73,762	0.0008	0.0843	69	3.458	93969	0.98	0.2877	0.647	-

### Russia 1995

sample	
mean	obs
61,240	2419

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-91.02	60,005	0.0012	0.1296	60	58927	1.018	-	-	-	0.497
lognormal	-160.72	72,225	0.0037	0.2233	221	10.48	1.19	-	-	-	0.600
Weibull	-91.24	60,059	0.0012	0.1321	60	0.999	60032	-	-	-	0.500
Dagum	-79.52	66,192	0.0008	0.0963	35	2.166	75201	0.444	-	-	0.549
Beta 1	-91.02	60,011	0.0012	0.1296	60	5E+11	1.018	9E+06	-	-	0.497
Beta 2	-82.51	61,555	0.0009	0.1072	42	3E+05	1.152	6.537	-	-	0.518
GG	-85.34	60,704	0.0010	0.1180	48	0.747	27849	1.638	-	-	0.510
Singh-Maddala	-81.51	61,965	0.0008	0.1021	40	1.126	2E+05	4.203	-	-	0.521
GB1	-85.34	60,703	0.0010	0.1180	48	0.747	2E+12	1.638	756662	-	0.510
GB2	-79.05	64,316	0.0008	0.0906	35	1.814	86580	0.547	1.3876	-	0.537
GB	-78.93	60,830	0.0008	0.0884	34	2.055	70165	0.975	0.4771	0.952	-

### Slovakia 1992

sample	
mean	obs
834	11439

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-140.79	824	0.0006	0.0892	129	238.8	3.453	-	-	-	0.293
lognormal	-232.18	841	0.0014	0.1349	312	6.573	0.569	-	-	-	0.312
Weibull	-275.33	820	0.0018	0.1447	399	2.001	925.7	-	-	-	0.293
Dagum	-216.26	847	0.0012	0.1331	283	3.742	925.9	0.592	-	-	0.311
Beta 1	-140.79	824	0.0006	0.0892	129	9E+08	3.453	4E+06	-	-	0.293
Beta 2	-138.87	826	0.0006	0.0871	125	11902	3.661	53.75	-	-	0.295
GG	-137.54	826	0.0005	0.0863	122	0.836	123.1	4.814	-	-	0.295
Singh-Maddala	-168.45	830	0.0008	0.1032	186	2.433	1192	2.485	-	-	0.299
GB1	-134.44	825	0.0005	0.0832	116	0.112	8321	59.46	18.655	-	-
GB2	-137.54	826	0.0005	0.0863	122	0.836	4E+08	4.814	286871	-	0.295
GB	-134.44	825	0.0005	0.0832	116	0.112	8321	0	59.464	18.65	-

### Spain 1980

sample	
mean	obs
669,912	18803

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-516.38	652,948	0.0024	0.1729	873	3E+05	2.59	-	-	-	0.334
lognormal	-1058.45	680,505	0.0049	0.2322	2255	13.2	0.68	-	-	-	0.369
Weibull	-611.22	648,783	0.0030	0.1971	1070	1.725	7E+05	-	-	-	0.331
Dagum	-300.68	662,885	0.0012	0.1246	438	3.569	8E+05	0.506	-	-	0.343
Beta 1	-516.38	652,990	0.0024	0.1729	873	4E+11	2.589	2E+06	-	-	0.334
Beta 2	-516.36	653,035	0.0024	0.1726	873	2E+08	2.597	738.7	-	-	0.334
GG	-509.02	651,640	0.0024	0.1757	853	1.145	4E+05	2.044	-	-	0.332
Singh-Maddala	-395.14	658,706	0.0017	0.1385	637	2.117	1E+06	2.529	-	-	0.338
GB1	-509.02	651,648	0.0024	0.1757	853	1.144	2E+10	2.045	206945	-	0.332
GB2	-272.21	676,894	0.0010	0.1202	381	5.467	7E+05	0.313	0.518	-	0.356
GB	-254.51	641,864	0.0010	0.1115	345	6.95	7E+05	0.998	0.2436	0.35	-

## Spain 1990

<i>sample</i>	
<i>mean</i>	<i>obs</i>
1,823,374	15526

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-389.47	1,769,559	0.0020	0.1555	635	7E+05	2.612	-	-	-	0.333
lognormal	-858.98	1,844,144	0.0048	0.2314	1779	14.2	0.677	-	-	-	0.368
Weibull	-439.59	1,757,885	0.0022	0.1784	754	1.742	2E+06	-	-	-	0.328
Dagum	-261.50	1,800,116	0.0011	0.1168	375	3.604	2E+06	0.496	-	-	0.343
Beta 1	-387.53	1,766,872	0.0020	0.1588	630	4E+07	2.525	61.15	-	-	0.331
Beta 2	-389.47	1,769,676	0.0020	0.1555	635	1E+13	2.611	2E+07	-	-	0.333
GG	-376.72	1,764,458	0.0019	0.1606	609	1.218	1E+06	1.854	-	-	0.329
Singh-Maddala	-312.82	1,780,341	0.0015	0.1297	481	2.075	3E+06	2.924	-	-	0.334
GB1	-376.72	1,764,466	0.0019	0.1606	609	1.218	3E+10	1.855	220821	-	0.329
GB2	-258.35	1,813,857	0.0011	0.1192	369	4.194	2E+06	0.417	0.7736	-	0.348
GB	-242.13	1,741,590	0.0010	0.1117	335	5.454	2E+06	0.993	0.3172	0.461	-

## Sweden 1967

<i>sample</i>	
<i>mean</i>	<i>obs</i>
23,531	4912

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-687.28	25,635	0.0108	0.3918	1830	17498	1.465	-	-	-	0.429
lognormal	-1576.77	33,425	0.0290	0.6036	7238	9.81	1.102	-	-	-	0.564
Weibull	-554.48	25,075	0.0068	0.3019	1740	1.398	27506	-	-	-	0.391
Dagum	-328.11	24,478	0.0051	0.2323	609	5.136	43352	0.198	-	-	0.384
Beta 1	-479.57	24,576	0.0087	0.3254	941	75592	1.115	2.315	-	-	0.387
Beta 2	-689.83	25,268	0.0101	0.3812	2011	4E+11	1.523	2E+07	-	-	0.422
GG	-387.50	24,460	0.0067	0.2818	711	2.953	47955	0.342	-	-	0.381
Singh-Maddala	-554.48	25,076	0.0068	0.3019	1740	1.398	3E+08	5E+05	-	-	0.391
GB1	-387.50	24,461	0.0067	0.2818	711	2.954	2E+06	0.342	99768	-	0.381
GB2	-293.38	25,188	0.0045	0.1943	526	17.71	34494	0.056	0.1767	-	0.399
GB	-293.43	25,202	0.0045	0.1943	525	19.34	34451	1	0.0515	0.161	-

## Sweden 1975

<i>sample</i>	
<i>mean</i>	<i>obs</i>
48,613	9535

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-277.37	65,898	0.0021	0.1636	426	34345	1.919	-	-	-	0.382
lognormal	-840.36	71,734	0.0076	0.3162	1908	10.84	0.831	-	-	-	0.443
Weibull	-218.12	65,312	0.0015	0.1440	288	1.504	72373	-	-	-	0.369
Dagum	-169.93	66,776	0.0010	0.1172	192	3.591	91300	0.371	-	-	0.383
Beta 1	-242.58	65,236	0.0018	0.1582	339	4E+05	1.667	8.547	-	-	0.370
Beta 2	-292.19	67,307	0.0024	0.1842	435	4E+11	1.787	1E+07	-	-	0.394
GG	-217.37	65,271	0.0015	0.1454	285	1.574	76684	0.93	-	-	0.369
Singh-Maddala	-215.26	65,483	0.0015	0.1356	287	1.553	4E+05	15.97	-	-	0.371
GB1	-217.37	65,272	0.0015	0.1454	285	1.574	2E+08	0.93	191480	-	0.369
GB2	-169.68	66,997	0.0010	0.1173	192	3.799	88675	0.349	0.8995	-	0.385
GB	-144.60	64,021	0.0007	0.0927	142	5.921	70943	0.993	0.2218	0.374	-



### Sweden 1981

<i>sample</i>	
<i>mean</i>	<i>obs</i>
82,115	8635

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-385.86	101,121	0.0035	0.2125	687	64274	1.573	-	-	-	0.416
lognormal	-1246.30	117,758	0.0137	0.4204	3117	11.2	0.973	-	-	-	0.509
Weibull	-263.15	99,796	0.0019	0.1444	436	1.387	1E+05	-	-	-	0.393
Dagum	-142.91	100,468	0.0008	0.0922	139	4.38	2E+05	0.24	-	-	0.394
Beta 1	-223.75	98,628	0.0018	0.1537	305	4E+05	1.23	3.183	-	-	0.387
Beta 2	-386.06	100,823	0.0034	0.2110	695	3E+12	1.586	4E+07	-	-	0.415
GG	-172.59	98,887	0.0012	0.1229	199	2.292	2E+05	0.475	-	-	0.385
Singh-Maddala	-263.15	99,797	0.0019	0.1444	436	1.387	2E+09	6E+05	-	-	0.393
GB1	-172.59	98,888	0.0012	0.1229	199	2.292	3E+07	0.475	88473	-	0.385
GB2	-142.67	100,746	0.0008	0.0934	138	4.667	2E+05	0.225	0.8824	-	0.395
GB	-138.33	97,803	0.0008	0.0839	130	6.104	1E+05	0.982	0.1713	0.463	-

### Sweden 1987

<i>sample</i>	
<i>mean</i>	<i>obs</i>
129,465	8658

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-667.13	146,993	0.0068	0.2987	1315	96894	1.517	-	-	-	0.422
lognormal	-1754.71	177,371	0.0207	0.5127	4465	11.56	1.021	-	-	-	0.530
Weibull	-482.61	144,490	0.0041	0.2210	980	1.397	2E+05	-	-	-	0.391
Dagum	-173.54	143,126	0.0011	0.1264	206	6.194	3E+05	0.157	-	-	0.379
Beta 1	-270.34	141,584	0.0023	0.1812	399	4E+05	1.086	1.911	-	-	0.378
Beta 2	-667.14	147,077	0.0068	0.2991	1313	6E+11	1.515	6E+06	-	-	0.423
GG	-198.93	141,964	0.0014	0.1477	256	3.56	3E+05	0.277	-	-	0.375
Singh-Maddala	-482.61	144,492	0.0041	0.2210	980	1.397	3E+09	7E+05	-	-	0.391
GB1	-198.93	141,962	0.0014	0.1477	256	3.56	6E+06	0.277	49755	-	0.375
GB2	-166.15	145,199	0.0010	0.1122	192	12.91	2E+05	0.074	0.3292	-	0.384
GB	-165.32	141,125	0.0010	0.1068	191	20.37	2E+05	1	0.0464	0.185	-

### Sweden 1992

<i>sample</i>	
<i>mean</i>	<i>obs</i>
189,352	10824

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-851.69	226,843	0.0076	0.3071	1661	2E+05	1.356	-	-	-	0.443
lognormal	-2334.97	287,266	0.0236	0.5395	5827	11.96	1.107	-	-	-	0.566
Weibull	-666.41	223,072	0.0051	0.2465	1383	1.304	2E+05	-	-	-	0.412
Dagum	-256.10	220,056	0.0017	0.1395	361	5.566	4E+05	0.164	-	-	0.398
Beta 1	-401.83	217,854	0.0033	0.2072	645	6E+05	1.01	1.984	-	-	0.397
Beta 2	-872.85	235,957	0.0085	0.3339	1596	5E+13	1.24	3E+08	-	-	0.459
GG	-298.11	218,205	0.0022	0.1639	441	3.239	5E+05	0.285	-	-	0.394
Singh-Maddala	-666.41	223,076	0.0051	0.2465	1383	1.304	9E+09	9E+05	-	-	0.412
GB1	-298.11	218,206	0.0022	0.1639	441	3.238	2E+07	0.285	99236	-	0.394
GB2	-245.12	223,510	0.0016	0.1235	341	10.2	4E+05	0.088	0.3819	-	0.405
GB	-245.04	218,252	0.0016	0.1227	341	11.04	4E+05	0.998	0.0811	0.337	-

## Sweden 1995

<i>sample</i>	
<i>mean</i>	<i>obs</i>
203,854	11655

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-1244.39	241,411	0.0107	0.3763	2569	2E+05	1.026	-	-	-	0.495
lognormal	-3410.68	379,192	0.0329	0.6371	9352	11.88	1.387	-	-	-	0.673
Weibull	-1179.37	238,273	0.0088	0.3359	2846	1.105	2E+05	-	-	-	0.466
Dagum	-424.18	228,174	0.0031	0.1946	713	5.866	5E+05	0.128	-	-	0.436
Beta 1	-599.14	227,535	0.0048	0.2520	1059	7E+05	0.784	1.554	-	-	0.435
Beta 2	-1244.45	241,904	0.0107	0.3770	2560	1E+13	1.023	6E+07	-	-	0.496
GG	-466.70	226,850	0.0035	0.2154	794	3.725	5E+05	0.201	-	-	0.432
Singh-Maddala	-1179.38	238,281	0.0088	0.3359	2846	1.104	6E+10	1E+06	-	-	0.466
GB1	-466.70	226,849	0.0035	0.2154	794	3.724	1E+07	0.201	69479	-	0.432
GB2	-399.51	233,621	0.0028	0.1730	665	18.85	4E+05	0.039	0.1882	-	0.446
GB	-398.88	223,702	0.0028	0.1705	664	24.96	4E+05	1	0.0296	0.134	-

## Switzerland 1982

<i>sample</i>	
<i>mean</i>	<i>obs</i>
56,036	5570

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-455.59	58,895	0.0073	0.3061	832	23937	2.46	-	-	-	0.342
lognormal	-807.36	62,749	0.0124	0.4058	2127	10.79	0.724	-	-	-	0.391
Weibull	-416.35	58,280	0.0069	0.3012	698	1.721	65373	-	-	-	0.331
Dagum	-270.07	58,482	0.0037	0.2152	415	4.021	75326	0.412	-	-	0.334
Beta 1	-444.84	58,557	0.0073	0.3083	778	5E+05	2.228	17.09	-	-	0.336
Beta 2	-455.59	58,903	0.0073	0.3061	832	1E+11	2.459	5E+06	-	-	0.342
GG	-413.93	58,362	0.0068	0.2983	699	1.573	58175	1.164	-	-	0.331
Singh-Maddala	-371.64	58,901	0.0054	0.2623	666	2.015	1E+05	3.529	-	-	0.333
GB1	-413.94	58,362	0.0068	0.2983	699	1.573	1E+08	1.165	169119	-	0.331
GB2	-204.79	60,995	0.0025	0.1591	279	12.94	61597	0.121	0.2014	-	0.360
GB	-204.79	60,995	0.0025	0.1591	279	12.94	61597	1	0.1211	0.201	-

## Switzerland 1992

<i>sample</i>	
<i>mean</i>	<i>obs</i>
75,080	4486

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-370.90	72,146	0.0068	0.2929	683	36580	1.972	-	-	-	0.377
lognormal	-819.48	80,722	0.0172	0.4461	2098	10.94	0.853	-	-	-	0.453
Weibull	-273.01	71,076	0.0045	0.2269	452	1.595	79250	-	-	-	0.353
Dagum	-181.71	70,969	0.0027	0.1677	232	4.577	1E+05	0.28	-	-	0.354
Beta 1	-288.49	70,636	0.0052	0.2572	454	3E+05	1.538	4.004	-	-	0.354
Beta 2	-370.90	72,145	0.0068	0.2929	683	5E+11	1.972	1E+07	-	-	0.377
GG	-240.40	70,585	0.0041	0.2298	349	2.306	1E+05	0.568	-	-	0.350
Singh-Maddala	-273.01	71,076	0.0045	0.2269	452	1.595	2E+08	3E+05	-	-	0.353
GB1	-240.40	70,587	0.0041	0.2297	349	2.306	1E+07	0.569	66930	-	0.350
GB2	-156.25	73,165	0.0021	0.1447	179	12.3	87239	0.102	0.2379	-	0.373
GB	-151.24	69,154	0.0020	0.1299	169	16.12	84322	1	0.0775	0.166	-

### United States 1969

sample	
mean	obs
9,816	9687

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-507.46	9,681	0.0043	0.2447	950	5050	1.917	-	-	-	0.382
lognormal	-1382.90	10,795	0.0135	0.4183	3378	8.918	0.859	-	-	-	0.456
Weibull	-321.88	9,549	0.0024	0.1721	542	1.562	10625	-	-	-	0.358
Dagum	-170.59	9,577	0.0010	0.1091	193	4.728	15143	0.257	-	-	0.360
Beta 1	-322.26	9,473	0.0027	0.1885	500	32748	1.473	3.618	-	-	0.357
Beta 2	-509.88	9,605	0.0042	0.2380	985	7E+10	1.972	1E+07	-	-	0.377
GG	-245.39	9,484	0.0018	0.1575	342	2.376	15373	0.532	-	-	0.354
Singh-Maddala	-321.88	9,549	0.0024	0.1721	542	1.562	5E+07	5E+05	-	-	0.358
GB1	-245.39	9,484	0.0018	0.1575	342	2.376	2E+06	0.532	157264	-	0.354
GB2	-155.37	9,736	0.0009	0.0885	161	7.832	13103	0.151	0.4447	-	0.368
GB	-154.32	9,430	0.0008	0.0866	159	8.776	12684	0.998	0.135	0.364	-

### United States 1974

sample	
mean	obs
13,465	9366

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-408.43	13,314	0.0034	0.2106	723	7308	1.822	-	-	-	0.391
lognormal	-1204.81	14,898	0.0118	0.3896	2867	9.222	0.88	-	-	-	0.466
Weibull	-264.27	13,146	0.0019	0.1454	409	1.504	14567	-	-	-	0.369
Dagum	-144.99	13,230	0.0008	0.0957	141	4.446	20859	0.266	-	-	0.372
Beta 1	-267.81	13,047	0.0021	0.1693	390	49172	1.437	3.979	-	-	0.367
Beta 2	-408.87	13,265	0.0034	0.2076	733	1E+11	1.844	2E+07	-	-	0.389
GG	-207.91	13,067	0.0015	0.1432	267	2.19	20808	0.566	-	-	0.365
Singh-Maddala	-264.27	13,146	0.0019	0.1453	409	1.504	9E+07	5E+05	-	-	0.369
GB1	-207.91	13,067	0.0015	0.1432	267	2.19	4E+06	0.566	99764	-	0.365
GB2	-135.84	13,427	0.0007	0.0896	122	6.577	18308	0.176	0.5214	-	0.379
GB	-134.17	12,974	0.0007	0.0870	119	7.646	17355	0.996	0.1511	0.394	-

### United States 1979

sample	
mean	obs
19,666	11609

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-369.42	19,526	0.0023	0.1741	636	11182	1.746	-	-	-	0.398
lognormal	-1217.87	21,784	0.0093	0.3388	2802	9.592	0.891	-	-	-	0.471
Weibull	-251.07	19,318	0.0014	0.1327	374	1.447	21299	-	-	-	0.381
Dagum	-154.41	19,566	0.0006	0.0893	163	4.039	30075	0.289	-	-	0.386
Beta 1	-269.43	19,210	0.0016	0.1495	401	88273	1.446	5.199	-	-	0.379
Beta 2	-369.44	19,541	0.0023	0.1746	635	1E+11	1.742	9E+06	-	-	0.398
GG	-220.41	19,240	0.0012	0.1296	299	1.874	28256	0.671	-	-	0.377
Singh-Maddala	-251.07	19,318	0.0014	0.1328	374	1.447	1E+08	3E+05	-	-	0.381
GB1	-220.41	19,240	0.0012	0.1296	299	1.874	1E+07	0.671	113757	-	0.377
GB2	-148.44	19,825	0.0006	0.0877	150	5.484	26939	0.208	0.6013	-	0.392
GB	-146.57	19,118	0.0006	0.0842	146	6.58	25154	0.994	0.1728	0.43	-

### United States 1986

<i>sample</i>	
<i>mean</i>	<i>obs</i>
31,538	9093

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-263.29	31,053	0.0019	0.1662	408	18989	1.635	-	-	-	0.409
lognormal	-937.39	35,059	0.0088	0.3453	2171	10.03	0.927	-	-	-	0.488
Weibull	-186.99	30,738	0.0012	0.1230	238	1.388	33678	-	-	-	0.393
Dagum	-109.58	31,185	0.0004	0.0691	70	3.811	47727	0.297	-	-	0.399
Beta 1	-203.47	30,580	0.0014	0.1406	261	2E+05	1.392	5.882	-	-	0.392
Beta 2	-265.95	31,377	0.0020	0.1742	402	3E+11	1.587	1E+07	-	-	0.414
GG	-167.04	30,614	0.0011	0.1219	185	1.753	44232	0.696	-	-	0.390
Singh-Maddala	-186.99	30,738	0.0012	0.1230	238	1.388	3E+08	3E+05	-	-	0.393
GB1	-167.05	30,615	0.0011	0.1219	185	1.752	2E+07	0.696	34305	-	0.390
GB2	-103.19	31,724	0.0003	0.0583	57	5.424	42021	0.203	0.5617	-	0.408
GB	-101.34	30,358	0.0003	0.0523	53	6.537	39292	0.996	0.1679	0.404	-

### United States 1991

<i>sample</i>	
<i>mean</i>	<i>obs</i>
37,501	11544

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-281.29	37,295	0.0018	0.1500	425	24464	1.524	-	-	-	0.422
lognormal	-1100.98	42,538	0.0088	0.3307	2427	10.19	0.965	-	-	-	0.505
Weibull	-206.76	36,952	0.0011	0.1182	267	1.326	40167	-	-	-	0.407
Dagum	-137.27	37,674	0.0005	0.0763	120	3.628	58019	0.297	-	-	0.415
Beta 1	-217.60	36,735	0.0012	0.1331	284	2E+05	1.315	6.047	-	-	0.405
Beta 2	-281.29	37,294	0.0018	0.1500	425	1E+11	1.524	4E+06	-	-	0.422
GG	-184.01	36,802	0.0010	0.1147	215	1.667	53156	0.701	-	-	0.403
Singh-Maddala	-206.76	36,953	0.0011	0.1182	267	1.326	7E+08	4E+05	-	-	0.407
GB1	-184.01	36,802	0.0010	0.1147	215	1.667	5E+07	0.701	96096	-	0.403
GB2	-136.66	37,871	0.0005	0.0771	119	3.985	55379	0.269	0.8439	-	0.417
GB	-133.86	36,491	0.0005	0.0734	113	4.948	48349	0.986	0.2148	0.522	-

### United States 1994

<i>sample</i>	
<i>mean</i>	<i>obs</i>
43,099	44255

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini
gamma	-640.15	42,097	0.0012	0.1220	1160	28068	1.5	-	-	-	0.424
lognormal	-3469.39	47,841	0.0071	0.3116	8083	10.31	0.967	-	-	-	0.506
Weibull	-443.19	41,756	0.0008	0.0962	731	1.304	45242	-	-	-	0.412
Dagum	-229.76	42,774	0.0003	0.0579	281	3.421	63867	0.318	-	-	0.423
Beta 1	-506.30	41,605	0.0009	0.1079	848	3E+05	1.341	8.265	-	-	0.412
Beta 2	-640.40	42,033	0.0012	0.1211	1164	3E+11	1.506	1E+07	-	-	0.424
GG	-410.96	41,648	0.0007	0.0951	650	1.502	54724	0.8	-	-	0.410
Singh-Maddala	-443.20	41,756	0.0008	0.0962	731	1.304	5E+08	2E+05	-	-	0.412
GB1	-410.97	41,648	0.0007	0.0951	650	1.502	1E+08	0.801	103236	-	0.410
GB2	-228.30	42,967	0.0003	0.0592	277	3.685	61524	0.293	0.8759	-	0.425
GB	-226.10	41,678	0.0003	0.0592	272	4.093	56508	0.986	0.2626	0.672	-

## United States 1997

<i>sample</i>	
<i>mean</i>	<i>obs</i>
50,384	39815

Distribution	Log-L	Mean	SSE	SAE	$\chi^2$	Parameters					Gini	
gamma	-508.82	46,792	0.0010	0.1153	868	31124	1.503	-	-	-	-	0.424
lognormal	-2844.48	52,892	0.0065	0.2917	6509	10.41	0.961	-	-	-	-	0.503
Weibull	-380.13	46,445	0.0007	0.1031	586	1.3	50287	-	-	-	-	0.413
Dagum	-192.79	47,719	0.0003	0.0552	208	3.295	69153	0.336	-	-	-	0.427
Beta 1	-440.62	46,372	0.0009	0.1133	706	4E+05	1.379	11.22	-	-	-	0.414
Beta 2	-509.15	46,705	0.0010	0.1143	871	5E+11	1.511	2E+07	-	-	-	0.423
GG	-372.77	46,384	0.0007	0.1032	566	1.396	55761	0.892	-	-	-	0.412
Singh-Maddala	-376.84	46,530	0.0007	0.1002	584	1.321	7E+05	31.58	-	-	-	0.415
GB1	-372.77	46,384	0.0007	0.1032	566	1.396	4E+08	0.892	203027	-	-	0.412
GB2	-190.26	48,035	0.0003	0.0550	203	3.652	65797	0.3	0.8356	-	-	0.430
GB	-187.77	46,359	0.0003	0.0538	198	4.106	60423	0.988	0.2657	0.635	-	-

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